

Collaborative Research: Learning Discrete Mathematics and Computer Science via Primary Historical Sources

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<http://www.cs.nmsu.edu/historical-projects>

1 Historical Projects we plan to develop

- **Gödel's Incompleteness Theorems.** Kurt Gödel (1906–1978) is widely regarded as one of the most significant logicians of all time. His work has had enormous impact on the 20th century scientific and philosophical development. Gödel's celebrated Incompleteness Theorems [28, 29] were a fatal blow to Hilbert's Program, and have forever changed the foundations of mathematics. In a recent book [30] Rebecca Goldstein calls Gödel's Incompleteness Theorems “the third leg, together with Heisenberg's uncertainty principle and Einstein's relativity, of that tripod of theoretical cataclysms that have been felt to force disturbances deep down in the foundations of the ‘exact sciences.’” What are these great theorems then, and what difference do they really make? We plan to address these and related questions in the project. We will develop the technique of Gödel numbers [63], and prove that in every consistent and sufficiently expressive theory there are statements that are true, but are not provable from the axioms of the theory. Consequently, we obtain that even if a theory is rich enough to express its own consistency, it will *never* be able to prove it! The project is designed for courses in logic, foundations of mathematics, and theoretical computer science. Primary Author: PI Guram Bezhanishvili.