Teaching with primary historical sources: Should it go mainstream? Can it?*†‡

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Abstract

Many are now teaching mathematics directly with primary historical sources, in a variety of courses and levels. How far should this be taken? Should we adapt or redesign standard courses to a completely historical approach, chiefly from primary sources? If so, what are the obstacles to achieving this? Materials? Instructor training and attitudes? Class time? Textbooks? Classroom pedagogy? What should and can we do about such things? We attempt to provide answers to these questions, and illustrate with a sample student project based on Pascal's *Treatise on the Arithmetical Triangle* how numerous core course topics can be learned via a primary historical source.

1 Introduction

I am truly honored to be asked to speak on integrating the history of mathematics in mathematics education. Advocating the teaching of mathematics

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using history is presumably not very controversial at this conference, more like "preaching to the choir", as one says in English. But I wish to be somewhat provocative, perhaps even controversial, by suggesting a dream I have had for some time, that all students should learn the principal content of their mathematics directly from studying primary sources, i.e., from the words of the original discoverers or creators of new mathematics, as is done in the humanities, where students read the great original literature, not just about the great literature. In other words, I propose that we rebuild the entire mathematics curriculum at all levels around translated primary sources studied directly by our students. If you think this is extreme, then at least I am fulfilling the role of being a provocative speaker.

My belief that we should and can aim for a mathematics curriculum that is rich throughout in primary sources has developed only very slowly from my own experiences in the past twenty years. First I would like to describe this personal evolution, because it reflects very clearly some of the important challenges involved in implementing my dream.

2 A personal odyssey as an illustration of issues

First I codeveloped two one-semester courses for beginning and advanced undergraduate university students, based entirely on primary historical sources. Somewhat ironically, I was motivated by William Dunham's description of a great theorem enrichment course for teachers in which he rewrote the original source material in his own words, but I and my collaborator Reinhard Laubenbacher decided to skip the rewriting step and toss the original sources at our students, partly because it seemed like too much work to rewrite things; of course in retrospect this became my chief pedagogical goal, to have students read original sources themselves, the only compromise being translation into English. These courses each follow several great mathematical themes and problems through millennia via primary sources. The courses have been continually successful now for two decades, and have led to two books (Laubenbacher & Pengelley 1999) (Knoebel et al 2007) each with multiple chapters built entirely around primary sources, from which different chapters can be taught in different incarnations of the course (Pengelley 1999). They fully embody my vision of courses and books in which primary

sources are the principal objects of study and learning.

However, while they are solid mathematics courses, they are not focused on syllabi in the standard curriculum, i.e., they do not fall into the category of a course in calculus, or discrete mathematics, or real analysis, or abstract algebra, or the many other compartmentalized topics in the typical institutional curriculum. They were instead designed around historical development of great ideas viewed through primary sources, not just a purely modern vision of mathematics; and they are flexible, with different subjects covered in different semesters. In other words, they were designed and implemented in total freedom, rather than under the constraint of an existing course syllabus. So while these courses are taken by many university mathematics students as an elective, and by students studying other disciplines, they are not in the "mainstream" of the curriculum. Moreover, students and colleagues alike tend to consider them as "history of mathematics" courses, simply because no other mathematics courses have any meaningful history in them. Together these features leave the two courses somewhat outside the main path of a standard undergraduate mathematics student's degree coursework, and hinder the adoption of the courses elsewhere.

At about the same time, I also became heavily involved in collaborative developing, teaching, and publishing of student projects for calculus courses (Lakey & Pengelley 1993). While these were not historical in nature, this began my very slow process of moving away from lecturing in regular teaching, towards a more student-centered, problem-driven classroom, which I personally find prerequisite to engaging primary historical sources as the principal objects of study. Only by combining student project activity with an active classroom and historical materials have I ultimately managed to even begin building a standard curriculum around primary sources. I have recently written on my current thoughts (Pengelley 2008) about creating a classroom dynamic in which students are engaged in high-level active work, rather than listening to lecture.

Then about thirteen years ago, I cocreated a graduate level mathematics course on the role of history in teaching mathematics, in which each graduate student develops a written teaching module based on historical material. While this is a successful mathematics education graduate course, it is a course on mathematics education, not a mathematics course based on historical material.

Around this time I also made my first attempt to inject a primary historical source in a substantial way into a regular course, namely Arthur Cayley's

first paper on group theory in an abstract algebra course when students first encounter groups (Pengelley 2005). I began to realize that my students could benefit tremendously from having their very first encounters with the notion of an abstract group be through the wonderful mathematics emerging in the nineteenth century that motivated Cayley to define and develop the abstract idea and first steps towards a theory of groups. This was my first indication that one can very profitably simply start using primary sources as key documents of study in a regular course, without dramatically changing the "content" of the course.

This idea has expanded greatly during recent years into an increasing collaboration with an expanding circle of colleagues. Some of us had experience both teaching with primary sources and with designing substantial student projects, which engage students in large multi-step assignments and written reports on their investigation, and which may last from one to several weeks at a time. We decided to combine these pedagogical approaches, and to focus on regular course content on the discrete side of mathematics, very broadly conceived. So with support from the U.S. National Science Foundation, I am now part of a team of seven faculty with additional collaborating writers and testers at numerous institutions, who are developing, testing, and evaluating student projects based on primary historical sources, for teaching regular course syllabi in discrete mathematics, abstract algebra, graph theory, combinatorics, logic, and computer science (e.g., courses on algorithms or automata theory). We hope also that a useful statistical evaluation of the effects of our historical projects will emerge from the nature and scope of this endeavor. Details of our 20–30 student projects based on primary sources, some completed, tested, and published, some yet to be written, are available in a resource book (Barnett et al 2009) and at our web sites (Bezhanishvili et al 2003) (Barnett et al 2008). Below I will use one of these projects to illustrate how I believe primary sources can be central to the curriculum.

Most recently, in teaching an upper undergraduate level geometry course, I realized that I could have students learn most of the course content on the hyperbolic non-Euclidean plane from the original sources by Euclid, Legendre, Lobachevsky, and Poincaré presented in the geometry chapter of my first book (Laubenbacher & Pengelley 1999) of annotated primary sources, so these pre-prepared primary sources fit well and easily into the course.

I finally feel I am on a route towards a standard course curriculum in which primary historical sources play a core role, but you can see that it has been a long, slow road, that I am benefiting from working in collaboration

with others, and that so far I still only have a part of some courses based this way. Nonetheless, I now actually see that this could grow into courses built entirely around primary sources. Of course I realize that I am only one of many around the world who are working to incorporate primary sources in key ways into the regular curriculum, and I would like to acknowledge and applaud everyone else's efforts as well; this gives me the inspiration of community to continue, and I hope that together we may have an impact. My intent in this section has simply been to show by example what some of the challenges are in basing courses on primary sources, but that it may be possible.

3 Motivations: why or why not?

Why should we use primary sources at the foundations of our teaching? Or why not?

The reasons for doing this have already been enunciated by many others over the years, but I will merely mention here motivation and deep connections along time, understanding essence, origin, and discovery, mathematics as a humanistic endeavor; practice moving from verbal to modern mathematical descriptions; reflection on present-day standards and paradigms; participating in the process of doing mathematics through experiment, conjecture, proof, generalization, publication and discussion; more profound technical comprehension from initial simplicity; also dépaysement (disorientation, cognitive dissonance, multiple points of view); and a question-based curriculum that knows where it came from and where it might be going. Questions before answers, not answers to questions that have not been asked. In (Barnett et al in press) from this volume, we illustrate specifically how pedagogical design principles like these can be built into student projects based on primary sources.

On the other hand, one can think of reasons why teaching with primary sources at the core might not be good to aim for. In his intentionally "devil's advocate" article (Siu 1995), Man-Keung Siu lists possible unfavourable factors, some of which could apply to primary sources. There are first some pedagogical ones: How can you set questions on it on a test?; It can't improve the student's grade; Students don't like it; Students regard it as just as boring as the subject mathematics itself!; Students do not have enough general knowledge of culture to appreciate it!; Progress in mathematics is to

make difficult problems routine, so why bother to look back?; What really happened can be rather tortuous. Telling it as it was can confuse rather than enlighten!; Does it really help to read original texts, which is a very difficult task?; Is it liable to breed cultural chauvinism and parochial nationalism?; Is there any empirical evidence that students learn better when history of mathematics is made use of in the classroom?

I now have enough experience actually teaching with primary sources to say that I personally have found all these concerns to be either untrue or irrelevant with my students and my chosen primary sources. I could elaborate and explain, but here I will only affirm my clear experience that with carefully selected and prepared primary source material, and the right pedagogical method in the classroom, these objections or concerns can and should be rejected. Thank you for raising them, Man-Keung!

Man-Keung also listed concerns that are logistical in nature, and I will address these next.

4 Logistical obstacles

Man-Keung Siu's logistical concerns about using history certainly can apply to teaching with primary sources:

- There is a lack of resource material on it!
- There is a lack of teacher training in it!
- I have no time for it in class!

The good news is that the first two concerns are the things we can all work on, and doing so will influence the third!

4.1 Is there a lack of resource material?

Yes, but the availability of published primary sources and translations in all aspects of mathematics has been growing at great speed in the past few decades, thanks to the work of many wonderful people, and this work we should all continue. One resource bibliography for finding available primary source material is (Pengelley 2003). It would be wonderful to have a continually updated central listing of these sources. Providing such a central resource

online is something incredibly useful that HPM could sponsor, and I believe it is necessary to widespread adoption of teaching with primary sources. I will address the important issue of convenient packaging of primary sources for teaching below.

4.2 Instructor training, motivation?

Is there a lack of teacher training in using primary sources? Yes, of course, and this challenge can and should be relieved by more formal training opportunities, which is another task for us. But the question, I think, really hints at a deeper issue. How do we interest other instructors in teaching with history, and in particular in using primary sources? It will only be through instructors' desire to teach with history that it will happen, not by coercion, since mathematics instructors like to make their own pedagogical decisions. Since I believe that enticement is the only way, we should entice with wonderful source material packaged to make instructors salivate at the idea of learning and teaching with them. Some teachers want or need prepared guiding materials in the form of textbooks or projects, while others like to create their own. So I believe that the solution lies in providing a variety of packaging for primary source materials, and flexibility in how they can be used, along with our own leading by example in our teaching. Let us discuss packaging.

4.3 Packaging into textbooks and projects versus time and pedagogical style

In many parts of the world, textbooks are the driving force behind curriculum and pedagogy, whether we like it or not. So I believe that success in attracting others to teaching mainly with primary sources will require us to create textbooks that have this as their theme. But student projects are also playing a more substantial role in teaching these days, so projects based on primary sources can complement or even supplant portions of a standard textbook, and thus play an intermediate role as stepping stones in the direction I am advocating. This is the approach I am currently working on, as mentioned earlier (Bezhanishvili et al 2003) (Barnett et al 2008) (Barnett et al 2009). In fact, a course could be built entirely on a sequence of projects based on primary sources, and I am working in that direction.

These issues cannot be divorced from the remaining question of whether there is time in class for teaching with historical sources. My personal experience is a resounding yes, there is time, and I constantly become stronger in that conviction, by changing my teaching in two respects.

First, one can move entirely away from lecturing, whether using historical sources or not, by having students first do advance reading of all new material at home, and working and writing profitably about it, entirely before initial class contact with the material. This makes lecture totally unnecessary and unfruitful, and means that class time is spent on student work and interaction with the instructor and each other, and some whole class discussion, that already starts at a higher level; the time thus saved and redirected from lecture is enormous. I have written about the details of how I implement this (Pengelley 2008).

Second, one can implement learning from primary sources through projects in such a way that it literally takes over core topics from the textbook, i.e., one can find and develop primary sources to teach core material of the course. Then the textbook becomes at most ancillary, perhaps a source of modern notation, extra exercises, and an alternative, more modern point of view. The time otherwise spent with the textbook will instead be spent learning the same material from primary sources, and the textbook becomes a supplement, not the other way around. I would like to elaborate on one example of this.

5 A sample project: Pascal on induction and combinatorics

To see a detailed example of how core syllabus material can be taught directly from a historical project, and its effect on students, consider an introductory discrete mathematics course intended to have students start learning to make proofs in mathematics, in which some key content is to learn mathematical induction as a proof technique, and to become comfortable with index notation, binomial coefficients, combination numbers, factorials, and some elementary number theory. I have combined all these core topics in a three week class project (Barnett et al 2009) (Bezhanishvili et al 2003) centered on Blaise Pascal's *Treatise on the Arithmetical Triangle* (Pascal 1991). This large student project is pedagogically analyzed in some detail in (Barnett et

al in press) elsewhere in this volume. Here I present just a few key excerpts, intending only to demonstrate the power of the primary source for covering core material, and the kinds of challenges I give students to achieve this.

Pascal's treatise expounds the principle of mathematical induction, and his triangle leads into combinatorics. In fact this is the first place in the mathematical literature where the principle of mathematical induction is enunciated so completely and generally, as a means of establishing the indefinite persistence of an observed pattern.

After a good bit of historical background and context, students begin studying Pascal's highly verbal definitions for the triangle. Since Pascal's work involves no index notation, students learn naturally about double-indexing from translating Pascal's description into modern terminology. The project continues with exercises for students based on the first several of Pascal's 23 *consequences* (theorems) and his proofs, connecting them to modern notation, indexing, summation notation, terminology, and the adequacy of Pascal's proofs by iteration or generalizable example. Pascal's consequences actually ease slowly and totally naturally into the concept of proof by mathematical induction in order to prove symmetry in the triangle, allowing the concept of induction to evolve in students minds, rather than being presented abstractly out of nowhere. Pascal proves his claims, even his mathematical induction, by generalizable example, largely because he has no indexing notation to deal conveniently with arbitrary elements. Having students make all this precise in full generality with modern notation enables them to begin to think in terms of induction before it is formally introduced, and to powerfully appreciate the efficacy of indexing notation.

The crowning consequence in Pascal's treatise is the twelfth, in which Pascal derives a formula for the ratio of consecutive numbers in a base. From this he will obtain an elegant and efficient "closed" formula for all the numbers in the triangle, a powerful tool for much future mathematical work. And it is right here that Pascal enunciates the general proof principle we call induction. Again we ask students to translate Pascal's proof by generalizable example into a modern and completely general proof. This is far from trivial, and even involves an understanding of a property of proportions that is largely lost today. We highlight the following excerpt from the middle of the project, consisting of primary source material and exercises for students, to illustrate the level of challenge and richness of content of the source for teaching core course material.

Twelfth Consequence

In every arithmetical triangle, of two contiguous cells in the same base the upper is to the lower as the number of cells from the upper to the top of the base is to the number of cells from the lower to the bottom of the base, inclusive.

Let any two contiguous cells of the same base, E, C, be taken. I say that

E :	C ::	2 :	3
the lower	the upper	because there are two cells from E to the bottom, namely E, H,	because there are three cells from C to the top, namely C, R, μ.

Although this proposition has an infinity of cases, I shall demonstrate it very briefly by supposing two lemmas:

The first, which is self-evident, that this proportion is found in the second base, for it is perfectly obvious that $\varphi : \sigma :: 1 : 1$;

The second, that if this proportion is found in any base, it will necessarily be found in the following base.

Whence it is apparent that it is necessarily in all the bases. For it is in the second base by the first lemma; therefore by the second lemma it is in the third base, therefore in the fourth, and to infinity.

It is only necessary therefore to demonstrate the second lemma as follows: If this proportion is found in any base, as, for example, in the fourth, $D\lambda$, that is, if $D : B :: 1 : 3$, and $B : \theta :: 2 : 2$, and $\theta : \lambda :: 3 : 1$, etc., I say the same proportion will be found in the following base, $H\mu$, and that, for example, $E : C :: 2 : 3$.

For $D : B :: 1 : 3$, by hypothesis.

Therefore $\underbrace{D + B}_{E} : B :: \underbrace{1 + 3}_{4} : 3$

$E : B :: 4 : 3$

Similarly $B : \theta :: 2 : 2$, by hypothesis

Therefore $\underbrace{B+\theta}_{C} : B :: \underbrace{2+2}_{4} : 2$

But $\quad B : E :: 3 : 4$

Therefore, by compounding the ratios, $C : E :: 3 : 2$. Q.E.D.

The proof is the same for all other bases, since it requires only that the proportion be found in the preceding base,and that each cell be equal to the cell before it together with the cell above it, which is everywhere the case.

6. Pascal's Twelfth Consequence: the key to our modern factorial formula
   (a) Rewrite Pascal's Twelfth Consequence as a generalized modern formula, entirely in our $T_{i,j}$ terminology. Also verify its correctness in a couple of examples taken from his table in the initial definitions section.
   (b) Adapt Pascal's proof by example of his Twelfth Consequence into modern generalized form to prove the formula you obtained above. Use the principle of mathematical induction to create your proof.

From his Twelfth Consequence Pascal can develop a "formula" (essentially the modern factorial formula) for the numbers in the triangle, which can then be used in future work on combinatorics, probability, and algebra. In the following project excerpt, we have students follow Pascal's generalizable example to do so in modern form.

Problem

Given the perpendicular and parallel exponents of a cell, to find its number without making use of the arithmetical triangle.

Let it be proposed, for example, to find the number of cell ξ of the fifth perpendicular and of the third parallel row.

All the numbers which precede the perpendicular exponent, 5, having been taken, namely 1, 2, 3, 4, let there be taken the same number of natural numbers, beginning with the parallel exponent, 3, namely 3, 4, 5, 6.

Let the first numbers be multiplied together and let the product be 24. Let the second numbers be multiplied together and let the product be 360, which, divided by the first product, 24, gives as quotient 15, which is the number sought.

For ξ is to the first cell of its base, V, in the ratio compounded of all the ratios of the cells between, that is to say, $\xi : V$

in the ratio compounded of	$\xi : \rho,$	$\rho : K,$	$K : Q,$	$Q : V$
or by the twelfth consequence	$3 : 4$	$4 : 3$	$5 : 2$	$6 : 1$

Therefore $\xi : V :: 3 \cdot 4 \cdot 5 \cdot 6 : 4 \cdot 3 \cdot 2 \cdot 1$.

But V is unity; therefore ξ is the quotient of the division of the product of $3 \cdot 4 \cdot 5 \cdot 6$ by the product of $4 \cdot 3 \cdot 2 \cdot 1$.

N.B. If the generator were not unity, we should have had to multiply the quotient by the generator.

7. Pascal's formula for the numbers in the Arithmetical Triangle
   (a) Write down the general formula Pascal claims in solving his "Problem." Your formula should read $T_{i,j}$ = "some formula in terms of i and j." Also write your formula entirely in terms of factorials.
   (b) Look at the reason Pascal indicates for his formula for a cell, and use it to make a general proof for your formula above for an arbitrary $T_{i,j}$. You may try to make your proof just like Pascal is indicating, or you may prove it by mathematical induction.

The project continues on perfectly naturally to integrate combinatorics, the binomial theorem, Fermat's Theorem (proof by induction on the base using Pascal's formula for the binomial coefficients and uniqueness of prime

factorization), and to end with the RSA cryptosystem. This goes far beyond the historical source, but shows how the source serves as a natural foundation for the flow of important core topics.

I make one final comment on the efficacy of a core project like this. On part of a final exam I gave my students a choice between a proof by induction of a standard homework-like summation formula from their textbook or digesting, explaining, and adapting a modern proof by induction from a Consequence in Pascal's treatise that they had never seen before. Half the students chose to do new interpretation and modern proof work from the Pascal treatise!

6 Finale

I must end with an exhortation of one more reason to teach core material from primary sources: It is inspiring, fun, lively, rewarding and enriching for instructors as well as students. It will keep you happy, excited, and alive.

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