Networks and Spanning Trees

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Notes to the Instructor

The project is designed to motivate the modern definition of a "tree" found in textbooks covering graph theory, and then offer several applications of trees as well as one of the first algorithms for finding a minimal spanning tree. The term "tree" arises from the work of Arthur Cayley, whose enumeration of trees is discussed in short excerpts from "On the Theory of the Analytical Forms Called Trees" and "A Theorem on Trees." This is contrasted with Heinz Prüfer's counting of trees, although the word "tree" never appears in his work. Prüfer introduces the material via an applied problem, namely the counting of all possible railway networks satisfying certain properties. In hindsight, each of these networks represents a "labeled tree." Finally an efficient algorithm for finding a minimal spanning tree is studied from the original work of Otakar Borùvka, who likewise discusses the problem without use of the term "tree." Borùvka sought the most economical construction of an electrical power network across the rural region of Southern Moravia, now part of the Czech Republic. This problem can be understood today as finding the tree of shortest total edge length from all possible n^{n-2} labeled trees on n towns.

The project requires no prior knowledge of graph theory. It is designed primarily for an advanced undergraduate course in combinatorics, graph theory or algorithm design, although parts of the project could be used in an introductory discrete mathematics course. For an elementary course, the instructor may wish to omit the last section on Borùvka's algorithm, and concentrate on counting trees. For an upper-level course with students who may have seen some graph theory, Prüfer's and Borùvka's writings offer excellent applications of trees. For instructors seeking a hurried coverage of the project, study of Cayley's first paper "On the Theory of the Analytical Forms Called Trees" could be replaced with the simple statement that Cayley introduces the term "tree" in this paper.