

Sums of numerical powers in discrete mathematics: Archimedes sums squares in the sand

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Notes to the instructor

This project is the first in a sequence on the topic of sums of numerical powers.

The audience for this project is students of introductory discrete mathematics. The project is quite flexible, and the instructor should be able to pick and choose, if desired, from the various activities offered. For a shorter project the instructor can choose selectively from this module. Students may need substantial guidance with some parts, and the instructor should be sure to work through all the details before assigning any student work.

The goal is for students to learn many basic notations, techniques, and skills in the context of an historically and mathematically authentic big motivating problem with multiple connections to other mathematics. Hopefully this will be much more effective and rewarding than simply being asked to learn various skills for no immediately apparent application. Many of the techniques first introduced in a discrete mathematics course simply arise naturally as needed in this project, like reindexing summation notation, inequalities, and telescoping sums. Instead of separately covering various such topics and techniques, that class time could simply be spent on the project, and students will learn those things in the process.

The project also asks students to conjecture from patterns they generate, develop their mathematical intuition and judgement, and try proving their conjectures, i.e., putting students in the creative driver seat. The setting of sums of powers in the context of primary sources allows a richness of questions and interpretations, especially includes deep connections to geometry and the two-way interplay with calculus, as well as basic algebra and linear algebra, and a richness of proof techniques, including natural comparison of the efficacy of various proof methods.

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