Figurate numbers and sums of numerical powers: Fermat, Pascal, Bernoulli

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Notes to the instructor

This project is the third episode of four connected projects following the epic story of formulas for sums of powers from the Pythagoreans to Euler; it is a central theme in the development of discrete mathematics and combinatorics. The audience for this episode is students of intermediate discrete mathematics or combinatorics, and the episode connects sums of powers to figurate numbers, binomial coefficients, Pascal's triangle, and Bernoulli numbers. The project is quite flexible, and the instructor should be able to pick and choose, if desired, from the various activities offered. For a shorter project the instructor can choose selectively from this module; some exercises are marked as optional if they are not critical to later work. Students may need substantial guidance with some parts, and the instructor should be sure to work through all the details before assigning any student work.

The goal is for students to learn many basic notations, techniques, and skills in the context of an historically and mathematically authentic big motivating problem with multiple connections to other mathematics. Hopefully this will be much more effective and rewarding than simply being asked to learn various skills for no immediately apparent application. Many of the techniques and phenomena introduced in a discrete mathematics or combinatorics course simply arise naturally in this project, like recursive definitions, delicate work with summations and inequalities, counting and geometry, binomial coefficients and combination numbers, and proofs by mathematical induction. Instead of separately covering various such topics and techniques, that class time could simply be spent on the project, and students will learn those things in the process.

The project also asks students to conjecture from patterns they generate, develop their mathematical intuition and judgement, and try proving their conjectures, i.e., putting students in the creative mathematical driver seat in an authentic context. The setting of sums of powers in the context of primary sources allows a richness of questions and interpretations, especially includes deep connections to geometry and the two-way interplay with calculus, as well as basic algebra and linear algebra, and a richness of proof techniques, including natural comparison of the efficacy of various proof methods.

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