Applications of Boolean Algebra: Claude Shannon and Circuit Design

Janet Heine Barnett*

Notes to the instructor

This project is designed for an introductory or intermediate course in discrete or finite mathematics that considers boolean algebra from either a mathematical or computer science perspective. The project does assume some (very minimal) familiarity with the set operations of union and intersection. This pre-requisite material may be gained by completing the companion (Boole) project described below, through reading a standard textbook treatment of elementary set operations, or via a short class discussion/lecture.

Based on a award-winning paper by Claude Shannon, *A Symbolic Analysis of Relay and Switching Circuits*, this project begins with a concise overview of two historical antecedents to Shannon's work. The first of these is George Boole's original work on 'the logic of classes,' included in part to provide students with a connection to another concrete example of a boolean algebra on which they can draw; the second of these is Edward Huntington's work on the axiomatization of boolean algebras, included in part to emphasize to students the relationship between abstract axiomatic structures and concrete models as examples of those structures. Section 2 of the project introduces and develops the use of boolean expressions to represent parallel and series circuits. Within the concrete context of the 2-valued boolean algebra associated with these circuits, the standard properties of a boolean algebra are developed in this section; specific project questions in this section also provide students with practice in using these identities to simplify and manipulate boolean expressions. In Section 3, the concept of a 'disjunctive normal form' for boolean expressions is introduced in the context of circuit design.

Two other projects on boolean algebra are available as companions to this project, either or both of which could also be used independently of this project. The first companion project "Origins of Boolean Algebra in the Logic of Classes: George Boole, John Venn and C. S. Peirce," is suitable as a preliminary to either the Huntington project or to the Shannon project. Without explicitly introducing modern notation for operations on sets (until the concluding section), that project develops a modern understanding of these operations and their basic properties within the context of early efforts to develop a symbolic algebra for logic. By steadily increasing the level of abstraction, that project also lays the ground work for a more abstract discussion of boolean algebra as a discrete structure, and explores a variety of other mathematical themes, including the notion of an inverse operation, issues related to mathematical notation, and standards of rigor and proof.

The second companion project "Boolean Algebra as an Abstract Structure: Edward V. Huntington and Axiomatization" could be used either as a preliminary to or as a follow-up to the

*Department of Mathematics and Physics; Colorado State University-Pueblo; Pueblo, CO 81001 - 4901; janet.barnett@colostate-pueblo.edu.

Shannon project on circuit design. That project explores the early axiomatization of boolean algebra as an abstract structure, based on Huntington's 1904 paper *Sets of Independent Postulates for the Algebra of Logic*. In addition to introducing the now standard axioms for the boolean algebra structure, the project illustrates how to use these postulates to prove some basic properties of boolean algebras. Specific project questions also provide students with practice in using symbolic notation, and encourage them to analyze the logical structure of quantified statements. The project also examines Huntington's use of the two-valued Boolean algebra on $K = \{0, 1\}$ — first studied by George Boole in his work on the logic of classes — as a model to establish the *independence* and *consistency* of one of his postulate sets. The final section of the project discusses modern (undergraduate) notation and axioms for boolean algebras, and provides several practice exercises to reinforce the ideas developed in the earlier sections.

Implementation with students of any of these projects may be accomplished through individually assigned work, small group work and/or whole class discussion; a combination of these instructional strategies is recommended in order to take advantage of the variety of questions included in the project.