Capacity of Large-Scale Wireless Networks Under Jamming: Modeling and Analyses

Hong Huang[†], Yousef Jaradat[‡], Satyajayant Misra[*], Amjad Abu-Baker[◇], Rafael Asorey-Cacheda[♭], Reza Tourani[*], Mohammad Masoud[‡], and Ismael Jannoud[‡]

Abstract—Distributed jamming has important applications not only in the military context but also in the civilian context where spectrum sharing is increasingly used and inadvertent jamming becomes a reality. In this paper, we derive the capacity bounds of wireless networks in the presence of jamming. We show that when the density of jammers is higher than that of target nodes by a certain threshold, the capacity of wireless networks approaches zero as the numbers of target nodes and jammers go to infinity. This is true even when the total power of target nodes is much higher than that of the jammers. We provide the optimal communication schemes to achieve the capacity bounds. We also describe the power efficiency of wireless networks, showing that there is an optimal target node density for power-efficient network operation. Our results can provide guidance for designing optimal wireless networking protocols that have to deal with large-scale distributed jamming.

Index Terms—Capacity of wireless networks; denial of service attack; distributed jamming.

I. Introduction

There have been many studies on the capacity of wireless networks in a wide range of scenarios [2], [9]–[12]. However, there is no prior work on an important scenario: wireless networks capacity in the presence of distributed jamming. Specifically, this scenario refers to a large-scale wireless network with a large number of jammers distributed within the network. We contend that this scenario is relevant and important by considering two applications. First, in the future battlefield where two adversary forces meet, the wireless devices of one force may become distributed jammers to the opposite force, and the number of nodes involved can

† H. Huang is with the Klipsch School of Electrical and Computer Engineering, New Mexico State University, Las Cruces, NM, USA, (e-mail: hhuang@nmsu.edu).

◇ A. Abu-Baker is with the Department of Telecommunication Engineering, Yarmouk University, Irbid, Jordan, (e-mail: aabubaker@yu.edu.jo).

‡ Y. Jaradat, M.Masoud, and I. Jannoud are with the Department of Computer and Communications Engineering, Al-Zaytoonah University of Jordan, Amman, Jordan, (e-mail: y.jaradat@zuj.edu.jo; m.zakaria@zuj.edu.jo; ismael.jannoud@zuj.edu.jo).

♭ R. Asorey-Cacheda is with the Departamento de Enxeara Telemtica, Universidade de Vigo, (e-mail: rasorey@det.uvigo.es).

∗ S. Misra and R. Tourani are with the Department of Computer Science, New Mexico State University, Las Cruces, NM, USA, (e-mail: misra@cs.nmsu.edu; rtourani@cs.nmsu.edu).

be in the tens of thousands. Second, in today's world of spectrum sharing and proliferation of wireless devices speaking different protocols, large-scale distributed jamming can occur in inadvertent ways with varying degrees of severity. This is evident in the concerns in the cellphone industry about the potential interference in a large-scale deployment of femtocells [26], since the signals from femtocells and macrocells cause distributed jamming to each other in the cellphone network. In this paper, we consider jamming in the broader sense, which includes both deliberate jamming and inadvertent jamming (interference).

In our prior work [3], we demonstrated using percolation theory that the capacity of the wireless network is reduced dramatically as the number of jammers increases even when the total power of the jammers is held constant. In fact, we showed that the capacity of the wireless network goes through a phase transition from high capacity regime to low capacity regime when the number of jammers increases beyond a certain threshold, again with the total jamming power held constant. However, this prior work provides only qualitative analysis. In this paper, we aim to provide a quantitative analysis to relate the capacity of the wireless network to parameters of both the nodes and the jammers in the network.

According to Shannon's capacity formula, $C = W \log (1 + SINR)$, two factors determine the capacity of a single link: the bandwidth or the degree of freedom W and the ratio of signal to interference and noise (SINR). In a large-scale wireless network with distributed jamming, the number of nodes (referred to as *target nodes* henceforth to distinguish from jamming nodes) provides a measure of degree of freedom, since these target nodes can form a distributed multiple-input multiple-output (MIMO) antenna array to achieve capacity [7], [8], [15]. Assuming constant transmission power, the density of target nodes provides a measure of the degree of path loss or the received power; whereas the density of jammers provides a measure of the degree of interference. The densities of nodes are related to the distances between transmitters and receivers, and distances between jammers and receivers; thus the interplay between the densities of target nodes and jammers has a major impact on the capacity of the wireless networks. In this paper, we aim to extricate the relationship between the capacity of a wireless network and the parameters reflecting the nodes degrees of freedom, received power, and encountered interference.

Our findings reveal that the dominating factors impacting the capacity of large-scale wireless networks are the densities of target nodes and jammers. We show that *when the density of jammers is higher than that of target nodes by a certain threshold, the capacity of wireless networks approaches zero as the numbers of target nodes and jammers go to infinity, even when the total power of target nodes are much higher than the total power of the jammers.* This has serious implications on the practical deployment of large-scale networks.

Furthermore, we find that the dependency of the network power efficiency, which is defined as the network's transport capacity [2] per unit power consumed, on the target network density exhibits a *bifurcated behavior*: When the target network density is small, the network power efficiency is an increasing function of target network density. When the target network density goes beyond a threshold, the network power efficiency becomes a decreasing function of target network density. In other words, *there is an optimal target network density for which the power efficiency of the network is maximized.*

This paper makes *three contributions*: 1) We are the first to provide the capacity bounds of wireless networks in the presence of jamming. We discover the threshold phenomenon described in the previous paragraph, which is consistent with our findings in the prior work using percolation theory [3]. Also, our results reduce to the results in [15] when the jamming signal can be treated as regular noise in the so-called *noise-like* regime, the details of which will be provided later. 2) We provide optimal communication schemes for the target network to achieve the capacity bounds. 3) We describe the power efficiency of wireless networks, showing that there is an optimal target node density for power-efficient network operation. Our results can provide guidance for designing optimal wireless networking protocols that deal with large-scale distributed jamming.

The paper is organized as follows. In Section II, we describe related work. In Section III, we describe the model. In Section IV, we provide the formulation for the cut-set capacity bound. In Section V, we describe optimal communication schemes to achieve capacity. In Section VI, we describe the power efficiency of wireless networks. We conclude in Section VII.

II. RELATED WORK

There are three areas of related work: capacity of wireless networks, jamming in wireless networks, and capacity of wireless networks in the presence of eavesdroppers (jamming is active, whereas eavesdropping is passive), which are described below.

A. Capacity of Wireless Networks

The study of the transport capacity of large-scale wireless networks started in the seminal paper [2]. Many subsequent studies followed. Here we provide a brief outline of these studies to convey a sense of the scope without trying to be comprehensive. Upper bounds to the transport capacity in relation to graphic locations and power constraints of the nodes were provided in [6]. Departing from previous methods based on geometry or signal-to-noise ratio (SNR), information theoretical analyses were provided in [7], [8]. The transport capacity of wireless networks was studied over fading channels [9], in various path-loss attenuation regimes [10], [11], and in the fixed SNR regime [12]. The gap between the upper bound and the achievable capacity was closed using percolation theory in [13]. Capacity regimes of wireless networks with arbitrary size and densities were studied in [15].

B. Jamming in Wireless Networks

Much previous work on jamming was conducted in the military context [16]. Recently, studies on jamming in civilian wireless networks, especially wireless sensor networks, which are especially vulnerable because of the field deployment, began to appear in the literature [17], [24]. A game-theoretical analysis of a transmitter and a jammer transmitting to the same receiver was provided in [18]. Denial of service or jamming at the MAC layer was studied in [19], [20], [29]. A linear programming formulation and distributed heuristics were presented in [30] to maximize jamming impact on traffic flows with the jammer resource constraint. In the area of counter-measures to jamming, jamming detection and mapping in wireless sensor networks was studied in [21], [23]. Error correction codes were proposed as a counter-measure to jamming in [22]. Two methods to combat jamming, channel surfing and spatial retreats, were proposed in [25]. An optimization formulation and a heuristic for jamming and defense strategies were presented in [26]. A protocol called DEEJAM was proposed to combat jamming at the MAC layer in [27]. Cross-layer jamming detection and mitigation were studied in [28]. In [3], a new type of jamming using low-power jammers was proposed, which was shown to be very effective. In [4], a new dimension of jamming utilizing null frequencies in communications protocols was investigated.

C. Capacity of Wireless Networks in the Presence of Eavesdroppers

One of the major challenges facing wireless networks is the presence of eavesdroppers. Shannon initiated the study of secret communications [31], which was followed by extensions to noisy channels [32], broadcast channels [33], and Gaussian channels [34]. Recently, there has been a resurgence of research in wireless physical layer security [35]–[40], [43]. Closer to this work, the secrecy capacity of wireless networks was investigated in decentralized wireless networks [40], in stochastic wireless networks with collusion [41], and in broadcast channels [43]. Secrecy capacity scaling was studied in reference [42].
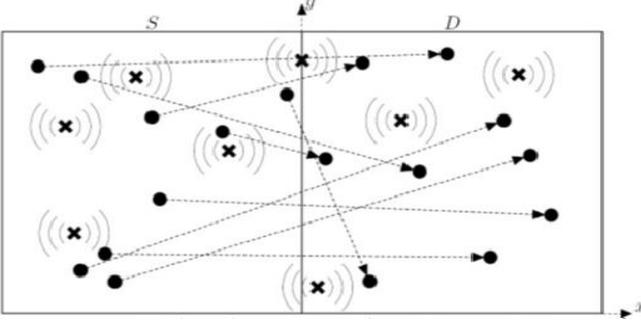
Fig. 1. The communication model, with the cut-set in the middle, target senders (solid circles) at the left side, target receivers (solid circles) at the right side, and jammers (crosses) deployed randomly throughout the network

Despite the diverse range of previous studies on jamming, there is no prior work on the capacity of wireless networks in the presence of jamming.

III. THE MODEL

We consider a wireless network with $8n_1$ target nodes and $2n_2$ jamming nodes uniformly randomly deployed in a rectangle of area $2A$ with length $2A^{1/2}$ and width $A^{1/2}$ respectively (refer Fig.1). For the target network, half of the nodes are randomly selected as the information sources, and the other half as the destinations. Each source in the target network randomly selects a destination and sends information. The total transmission power is n_1p_1 (it is assumed that all sources have equal transmission power), whereas the total jamming power is n_2p_2. We do not require jammers to have any kind of channel state information, but we assume the target transmitters have channel state information about their receivers by using standard estimation techniques [1]. We consider a cut length-wise in the middle of the target network. Asymptotically, with probability 1/2, a source node is at the left side of the cut. Further, with probability 1/4, the destination node of this source is at the right side, refer to Fig.1. Similar to [15], we study the cut-set capacity C of the target network, i.e., the sum of rates in bits/s/Hz across the cut from left to right, which is asymptotically equal to one fourth of the total throughput $R_{network}$ as n_1 approaches infinity since the network traffic goes through the cut with probability 1/4. In other words, the cut-set capacity is indicative of the total network throughput. In particular we are interested in the scaling law of C as n_1 approaches infinity. In other words, we are interested in the exponent e defined as

$$e \equiv \lim_{n_1 \to \infty} \frac{\log C}{\log n_1}. \tag{1}$$

The cut-set capacity is upper-bounded by the capacity of the MIMO channel between the target transmitters on the left side of the cut and the target receivers on the right side. Given the transmitted target signal vector X from the left side of the cut, and the jamming signal vector Z from both sides of the cut, the received signal vector Y on the right side of the cut can be expressed as

$$Y = H_1X + H_2Z + N_o, \tag{2}$$

where the individual components of X, Z, N_o are independent zero-mean circularly symmetric Gaussian target signals, jamming signals, and noises, with variances of p_1, p_2, and 1, respectively. Moreover, H_1 and H_2 are the $n_1 \times n_1$ and $n_1 \times 2n_2$ channel matrices for the signals received from the target nodes and the jammers, respectively, which are given below

$$H_{1,i,k} = \frac{\sqrt{G_1}e^{j\theta_{1,i,k}}}{d_{1,i,k}^{\alpha/2}}, \quad H_{2,i,k} = \frac{\sqrt{G_2}e^{j\theta_{2,i,k}}}{d_{2,i,k}^{\alpha/2}}, \tag{3}$$

where $d_{1,i,k}$ and $d_{2,i,k}$ are the distances between target transmitter k, jammer k and target receiver i, respectively; $\theta_{1,i,k}$ and $\theta_{2,i,k}$ are the corresponding phases; G_1 and G_2 are the respective gain coefficients, which we set to be 1 without loss of generality; and α is the path loss exponent. We assume a fast fading flat channel, and in such case $\theta_{1,i,k}$ and $\theta_{2,i,k}$ are independently and identically distributed uniformly in $[0, 2\pi]$. Similar assumptions are used in [14], [15].

We consider the network area scales with the number of target nodes in the following way: $2A = n_1^\nu$. This scaling relationship is a generalization of the special cases of the dense network where $2A = 1$ ($\nu = 0$), and of the extended network where $2A = n_1$ ($\nu = 1$), both of which are often referred to in the literature. More importantly, this scaling relationship allows the full exploration of capacity regimes from the high SINR regime implied by the dense network (higher density leads to shorter distance and higher received power) to the low SINR regime implied by the extended network. We use $d_{1,0}$ and $d_{2,0}$ to denote the network-wide average distances between any target node and its nearest target neighbor and jammer neighbor respectively. Because nodes are uniformly distributed we have

$$d_{1,0} = \sqrt{\frac{A}{n_1}} = \sqrt{n_1^{(\nu-1)}}, \; d_{2,0} = \sqrt{\frac{A}{n_2}} = \sqrt{\frac{n_1^\nu}{n_2}}. \tag{4}$$

Similar to [15], we introduce the exponent β so that $n_1^\beta \equiv d_{1,0}^{-\alpha}$, then we have $\beta = \alpha(1-\nu)/2$. Apparently β indicates the density of target nodes. Moreover, p_1 being constant, β indicates the level of signal-to-noise ratio (SNR) received from the nearest target nodes, the so-called short-range SNR. In the extended network, $\beta = 0$, and in the dense network $\beta = \alpha/2$.

In addition, we define the exponent γ so that $n_1^\gamma \equiv d_{2,0}^{-\alpha}$. So, γ indicates the density of the jammers and also the level of interference coming from the nearest jammer. The definition of γ uses n_1 rather than n_2 to put γ in the same scale as β. Also it simplifies the expression of the scaling exponent results. Using (4), we have

$$n_1^{\beta-\gamma} = \frac{d_{1,0}^{-\alpha}}{d_{2,0}^{-\alpha}} = \left(\frac{n_1}{n_2}\right)^{\alpha/2}. \tag{5}$$

From the last expression, we see that $\beta - \gamma$ is a function of the ratio of the numbers of target nodes and jammers in the network. In the special case where n_1/n_2 is a constant, $\beta - \gamma$ approaches zero as n_1 approaches infinity, indicating target nodes and jammers have similar density.

Finally, without loss of generality, we impose the constraint that the total power of the jammers is a fraction of the total power of the target nodes, i.e.,

$$n_2 p_2 = \rho n_1 p_1 \tag{6}$$

where $0 < \rho < 1$ is a small constant, say 0.01. In general, the capacity scaling results do not change as long as ρ is a constant. From (5) and (6), we have

$$\frac{p_1}{p_2} = \frac{n_1^{2(\gamma-\beta)/\alpha}}{\rho}. \tag{7}$$

IV. THE CUT-SET CAPACITY BOUND

We consider the ergodic cut-set capacity of the target network in a fast fading channel environment. According to information theory, the cut-set capacity is the maximum mutual information between X and Y on either sides of the cut and can be written as

$$C = \max I(X;Y) = \max[H(Y) - H(Y|X)]$$
$$= \max[H(Y) - H(Z + N_o)]. \tag{8}$$

The mutual information depends on the covariance matrices of the received signal Y and the jamming signal plus noise $Z + N_o$, which can be written as the following by using Eqn. (2)

$$R_Y = E[YY^H] = I + H_2 R_Z H_2^H + H_1 R_X H_1^H, \tag{9}$$

$$R_{Z+N_o} = E[(Z + N_o)(Z + N_o)^H] = I + H_2 R_Z H_2^H. \tag{10}$$

In the previous expressions, we have made the noise term to be an identity matrix without loss of generality. For given R_Y and R_{Z+N_o}, the mutual information in (8) is maximized when both Y and $Z + N_o$ are zero-mean circularly symmetric complex Gaussian random vectors [5], and the cut-set capacity in bits/s/Hz can be written as

$$C \le \max_{\substack{E[tr(R_X)] \le n_1 p_1 \\ E[tr(R_Z)] \le n_2 p_2}} E[\log \det(R_Y) - \log \det(R_{Z+N_o})]. \tag{11}$$

We provide a reduced form of the above capacity upper bound in Theorem 1.

Theorem 1: The capacity upper bound is given by:

$$C \le \int_0^{\sqrt{n_1}} \int_1^{\sqrt{n_1}} [\log(1 + c_2 p_2 n_1^\gamma + c_1 p_1 n_1^\beta x^{2-\alpha})$$
$$- \log(1 + c_2 p_2 n_1^\gamma)]dxdy, \tag{12}$$

where c_1 and c_2 are constants. $\qquad\square$
Proof: Refer to Appendix A. $\qquad\blacksquare$

Using Theorem 1, we obtain our main result as follows.

Theorem 2: The capacity scaling exponents of wireless networks in the presence of distributed jamming are those listed in Table I. $\qquad\square$
Proof: Refer to Appendix B. $\qquad\blacksquare$

Our results in Table I provide the expressions of the capacity scaling exponent e in terms of parameters α, β, and γ. Below, we provide some clarifying remarks for the information conveyed in Table I.

1) There are two operation regimes: *noise-like* and *jamming-dominant*. In the noise-like regime where $p_2 n_1^\gamma \le \infty$ as $n_1, n_2 \to \infty$, the jamming signal from nearest neighbors, which is $O(p_2 n_1^\gamma)$, can be bounded by a constant, which can be treated as effective noise for a node. The network behaves as if there exists only effective noise and the jamming does not exist. Our results in this regime revert to those in [15]. In the jamming-dominant regime where $p_2 n_1^\gamma \to \infty$ as $n_1, n_2 \to \infty$, the jamming signal from the nearest neighbors approaches infinity as n_1, n_2 approaches infinity. Thus, the noise is dominated by the jamming signal and can be ignored. This regime is new, we will focus our discussion on this regime in the following.

2) In the jamming-dominant regime, the scaling exponent is upper-bounded by 1, i.e., linear scaling is the best one can achieve.

3) In the jamming-dominant regime, the scaling exponent is linear and an increasing function of $\beta - \gamma$. According to (5), $\beta - \gamma$ is an increasing function of the ratio of the number of target nodes versus that of jammers. Thus, the capacity of wireless network is predominantly determined by the density ratio of target nodes versus jammers. Note that the capacity exponent e has nothing to do with ρ, the ratio of the total jamming power versus the total power of target network, as long as ρ is a constant.

4) In the jamming-dominant regime, the scaling exponent can be negative if $\beta - \gamma < (\alpha/2 - 2)(1 - 2/\alpha) = 3 - \alpha/2 - 4/\alpha$ when $2 \le \alpha < 3$, and if $\beta - \gamma < -1/2 + 1/\alpha$, when $\alpha \ge 3$. The above conditions constitute the *threshold* for degrading of network capacity in the presence of distributed jammers. In other words, *the capacity of the wireless network can go to zero as the number of nodes approaches infinity, as long as the jammer density is higher than the target node density by a certain threshold, even when the total transmitting power of the target nodes are much larger than that of the jammers.* This threshold phenomenon is consistent with our findings in the prior work using percolation theory [3], and it has grave implications to the practical deployment of target nodes and jammers.

In the following, we provide some concrete examples. We separate the discussion into two regimes: jamming-dominant regime and noise-like regime.

Jamming-dominant regime: When $\beta - \gamma \ge 0$, the exponent e lies in the range $[1/2, 1]$, with $e = 1$ (linear-scaling) when

TABLE I
SCALING EXPONENT OF WIRELESS NETWORKS IN THE PRESENCE OF JAMMING

Noise-like regime $p_2 n_1^\gamma \le \infty$ as $n_1, n_2 \to \infty$	$\beta \ge \alpha/2 - 1$		$e = 1$	
	$\beta < \alpha/2 - 1$	$\beta > 0$	$e = \begin{cases} 2 - \alpha/2 + \beta & 2 \le \alpha < 3 \\ 1/2 + \beta/(\alpha - 2) & \alpha \ge 3 \end{cases}$	
		$\beta \le 0$	$e = \begin{cases} 2 - \alpha/2 + \beta & 2 \le \alpha < 3 \\ 1/2 + \beta & \alpha \ge 3 \end{cases}$	
Jamming-dominant regime $p_2 n_1^\gamma \to \infty$ as $n_1, n_2 \to \infty$	$\beta - \gamma \ge \alpha/2 + 2/\alpha - 2$		$e = 1$	
	$\beta - \gamma < \alpha/2 + 2/\alpha - 2$	$\beta - \gamma > 0$	$e = \begin{cases} 2 - \alpha/2 + (\beta - \gamma)/(1 - 2/\alpha) & 2 \le \alpha < 3 \\ 1/2 + (\beta - \gamma)/(\alpha + 4/\alpha - 4) & \alpha \ge 3 \end{cases}$	
		$\beta - \gamma \le 0$	$e = \begin{cases} 2 - \alpha/2 + (\beta - \gamma)/(1 - 2/\alpha) & 2 \le \alpha < 3 \\ 1/2 + (\beta - \gamma)/(1 - 2/\alpha) & \alpha \ge 3 \end{cases}$	

$\beta - \gamma \ge \alpha/2 + 2/\alpha - 2$; and $e = 1/2$ (Gupta-Kumar-scaling), when $\alpha \ge 3$ and $\beta - \gamma = 0$ or α approaches infinity. For example, to achieve linear capacity scaling for $\alpha = 2$, we need to have $\beta - \gamma \ge 0$, which means the target network has an equal or higher node density than the jammer network. In contrast, for $\alpha = 4$, we need to have $\beta - \gamma \ge 1/2$, which, according to (5), translates to the requirement that the target network has a higher density than that of the jammer network.

When $\beta - \gamma < 0$, the exponent e lies in the range $[1/2 + \beta - \gamma, 1)$, with e approaching 1 when $\beta - \gamma$ approaches $\alpha/2 + 2/\alpha - 2$; and $e = 1/2 + (\beta - \gamma)/(1 - 2/\alpha)$ when $\alpha \ge 3$, where e drops below the Gupta-Kumar-scaling coefficient $(1/2)$.

Noise-like regime: This regime is similar to the jamming-dominant regime except γ disappears in the expressions. When $\beta \ge 0$, the exponent e lies in the range $[1/2, 1]$, with $e = 1$ (linear-scaling) when $\beta = \alpha/2 - 1$; and $e = 1/2$ (Gupta-Kumar-scaling), when $\beta = 0$ and $\alpha \ge 3$ or when α approaches infinity. For example, to achieve linear capacity scaling for $\alpha = 2$, we need to have $\beta \ge 0$, which requires the target network no less dense than that of the extended network $(2A = n_1)$. For $\alpha = 4$, we need to have $\beta \ge 1$, which requires that the area of the target network scales at least as the square root of the number of target nodes $(2A = n_1^{1/2})$, which is much denser than the extended network.

When $\beta < 0$, the exponent e lies in the range $[1/2 + \beta, 1)$, with e approaching 1 when β approaches $\alpha/2 - 1$; and $e = 1/2 + \beta$ when $\alpha \ge 3$.

V. OPTIMAL COMMUNICATION SCHEMES TO ACHIEVE CAPACITY BOUNDS

In this section, we consider communication schemes using distributed MIMO arrays to achieve network capacity in the jamming-dominant regime, i.e., $p_2 n_1^\gamma \to \infty$ as $n_1, n_2 \to \infty$, since the noise-like regime is equivalent to that without jamming, according to the discussion in the previous section, and it has been studied in [15]. Here, we modified the framework in [15] to take the contributions from jammers into account. The optimal communication schemes basically follow the approach in the proof of Theorem 2. We divide the network of $2n_1$ target nodes into $2k$ cells, with each cell containing m_1

target nodes. Thus, we have $k = n_1/m_1$, and each cell has the dimension of $\sqrt{Am_1/n_1} \times \sqrt{Am_1/n_1}$. Suppose there are n_1 source nodes, each sending one bit to their corresponding destination nodes. We assume each cell forms a distributed MIMO array and communicates with its neighbor cells in the following way. As show in Figure 2, the communications in the target network take place in three stages as follows.

- In the first stage, each source takes turns to broadcast m_1 bits in the local cell, one bit for each node in the cell. At the end of the stage, every node in the cell has the one bit for each node in the cell. Since there are m_1 nodes, each sending out one bit to $m_1 - 1$ nodes, this stage has a time complexity of $m_1(m_1 - 1) = O(m_1^2)$.
- In the second stage, bits are communicated using standard MIMO transmissions [14] between neighboring cells. It takes $O(k^{1/2})$ cell-hops for a bit in the source cell to reach the destination cell. For routing, we draw a straight line from the source cell to the destination cell; the cells intersecting the line are the sequence of hops used. Each time, only one pair of source and destination cells is involved in transmission and therefore there is no interference. Since there are k cells, each having to communicate m_1 bits over $O(k^{1/2})$, the time complexity is $O(k^{1/2}km_1) = O(k^{1/2}n_1)$.
- In the third stage, each node in the destination cells takes turns to broadcast its received bit to enable each node to decode the MIMO transmissions received in the second stage. Since there are m_1 nodes, each sending out one bit to $m_1 - 1$ nodes, this stage has the time complexity of $m_1(m_1 - 1) = O(m_1^2)$

Thus, the above communication scheme has a time complexity of $O(m_1^2 + k^{1/2}n_1)$. Note that the traditional multi-hop communication scheme is a special case of the above distributed MIMO scheme with $m_1 = 1$, and the hierarchical cooperation scheme in [14] is a special case where $m_1 = n_1$.

The MIMO sum rate $R_{cell}(m_1)$ between two neighbor cells, with m_1 target nodes each, can be considered as a scaled-down version of that of the original target network discussed in Section III and IV, except we have to replace n_1 with m_1. Applying Equation (27) and using the inequality $x \le m_1^{1/2}$
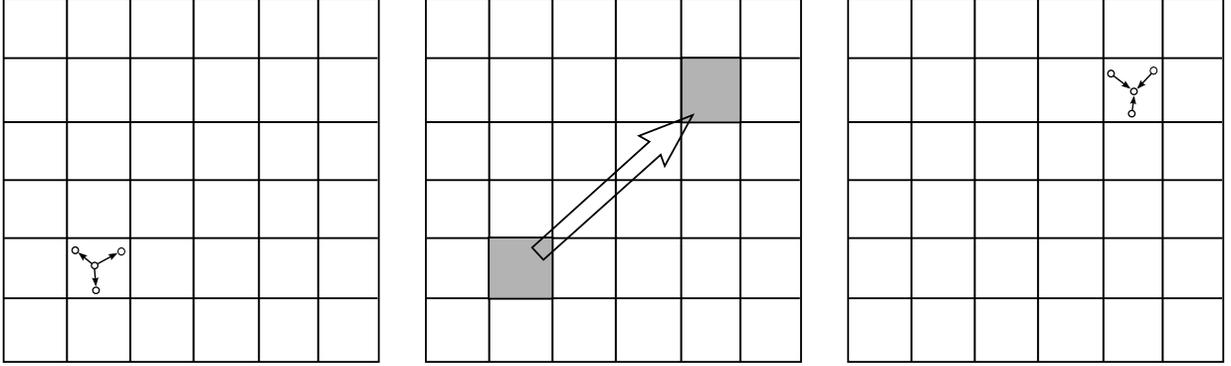
Fig. 2. The MIMO transmission is carried out in three phases. In Phase 1, bits are distributed within a cell to prepare for MIMO transmission. In Phase 2, multiple simultaneous MIMO transmissions occur between source and destination cells. In Phase 3, bits reach their destinations within the destination cell.

within a cell, the sum rate can be written as

$$R_{cell}(m_1) > c_7 m_1^{1-\epsilon} \log(1 + c_8 m_1^{1-\alpha/2} n_1^{(\beta-\gamma)(1-2/\alpha)}). \tag{13}$$

The above result is similar to the rate of the classical MIMO system with m_1 transmit and receive antennas, with $m_1^{-\epsilon}$ accounting for the overhead to set up the MIMO transmission. This rate per cell is shared by $k^{1/2}$ cells on average, which use the cell for relaying. Another way to put it is that it takes $k^{1/2}$ cell-hops on average for a packet to travel from the source cell to the destination cell. Therefore, each source cell can achieve a throughput of $R_{cell}/k^{1/2}$; and since there are k cells in the network, the throughput of the entire network can be written as

$$R_{network}(m_1) > c_7 k^{\frac{1}{2}} m_1^{1-\epsilon} \log(1 + c_8 m_1^{1-\frac{\alpha}{2}} n_1^{(\beta-\gamma)(1-2/\alpha)})$$
$$= c_7 n_1^{\frac{1}{2}} m_1^{\frac{1}{2}-\epsilon} \log(1 + c_8 m_1^{1-\frac{\alpha}{2}} n_1^{(\beta-\gamma)(1-2/\alpha)})$$
$$\equiv c_7 n_1^{\frac{1}{2}} m_1^{\frac{1}{2}-\epsilon} \log(1 + SINR_{cell}(m_1)). \tag{14}$$

In Equation (14), if $SINR_{cell}(m_1) > 1$ for all m_1, or $\beta-\gamma > \alpha/2 + 2/\alpha - 2$, the network throughput is an asymptotically increasing function of m_1 and is maximized by making $m_1 = n_1$, thus achieving linear scaling, which corresponds to the first row in Table I. If $SINR_{cell}(m_1) < 1$ for all m_1, or $\beta - \gamma < \alpha/2 + 2/\alpha - 2$ and $\beta - \gamma < 0$, using the fact that $\log(1 + x) < x$, Equation (14) can be approximated by

$$R_{network}(m_1) > c_9 m_1^{\frac{(3-\alpha)}{2}-\epsilon} n_1^{\frac{1}{2}+\frac{\beta-\gamma}{1-2\alpha}}. \tag{15}$$

Depending on whether $\alpha < 3$ or $\alpha > 3$, the network throughput is an increasing or decreasing function of m_1 and is maximized by making $m_1 = n_1$ or $m_1 = 1$, respectively, thus achieving the scaling laws in the last two rows in Table I. If $SINR_{cell}(m_1) > 1$ for some but not all m_1, or $\beta - \gamma > \alpha/2 + 2/\alpha - 2$ and $\beta - \gamma > 0$, we have two cases. In the case where $\alpha < 3$, using the approximation in Equation (15), it is clear to see that the network throughput is an increasing function of m_1 and is maximized by making $m_1 = n_1$, thus achieving the scaling law in the second

row in Table I. In the case where $\alpha > 3$, the network throughput is maximized around the value of m_1, where $SINR_{cell}(m_1) = 1$ is satisfied, or

$$m_1 = n_1^{\frac{2(\beta-\gamma)}{\alpha+4/\alpha-4}}. \tag{16}$$

Plugging (16) into (15) we have

$$R_{network}(m_1) > c_9 m_1^{-\epsilon} n_1^{\frac{1}{2}+\frac{(\beta-\gamma)}{\alpha+4/\alpha-4}} \tag{17}$$

which achieves the scaling law in the fifth row of Table I.

We summarize the optimal communication schemes to achieve capacity in Table II. Which communication scheme or cell size to use depends on the interplay between path loss, MIMO multiplexing gain, and the relative strength of the target and jamming signals. When path loss is small ($\alpha < 3$), network-wide MIMO transmission is used regardless of the other parameters. When path loss is large ($\alpha > 3$), network-wide MIMO transmission is not optimal and there is a certain optimal MIMO cell size to achieve capacity, including the case of an one-node cell which is the classical multi-hop scheme where the jamming signals dominates the target signals ($\beta - \gamma < 0$).

VI. POWER EFFICIENCY OF WIRELESS NETWORKS

In this section, we are concerned with power efficiency of wireless networks. For this purpose, inspired by the highly useful concept of work from physics, we define the communication work (W_c, referred to as transport capacity in [2]) as the product of the number of bits transported per second ($R_{network}$) in the network and the distance traveled by the bits (x):

$$W_c \equiv \int R_{network} \, dx. \tag{18}$$

We define the power efficiency (η) as the work performed per unit power consumed:

$$\eta \equiv \frac{W_c}{n_1 p_1}. \tag{19}$$

TABLE II
MIMO COMMUNICATION SCHEMES (CELL SIZE M1) TO ACHIEVE CAPACITY

$\beta - \gamma \geq \alpha/2 + 2/\alpha - 2$		$m_1 = n_1$	
$\beta - \gamma^+ < \alpha/2 + 2/\alpha - 2$	$\beta - \gamma > 0$	$m_1 = \begin{cases} n_1 & 2 \leq \alpha < 3 \\ n_1^{\frac{2(\beta-\gamma)}{\alpha+4/\alpha-4}} & \alpha \geq 3 \end{cases}$	
	$\beta - \gamma \leq 0$	$m_1 = \begin{cases} n_1 & 2 \leq \alpha < 3 \\ 1 & \alpha \geq 3 \end{cases}$	

We also define the exponent e_η associated with η as the following:

$$e_\eta \equiv \lim_{n_1, n_2 \to \infty} \frac{\log \eta}{n_1}. \tag{20}$$

The exponent e_η provides an asymptotic measure of the power efficiency of a communication scheme. A positive e_η indicates the network becomes more power-efficient as the number of target nodes increases, whereas a negative e_η indicates the network becomes less power-efficient as the number of target nodes becomes larger.

Since the cut-set capacity $(C = n_1^e)$ is asymptotically equal to one fourth of the network throughput $(R_{network})$ as we mentioned in Section II, and the distance traveled by the bits is of the order of $A^{1/2}$, using the fact $2A = n_1^\nu = n_1^{1-2\beta/\alpha}$, we have

$$e_\eta = \lim_{n_1, n_2 \to \infty} \frac{\log \frac{W_c}{n_1 p_1}}{\log n_1} = \lim_{n_1, n_2 \to \infty} \frac{\log \frac{n_1^e A^{\frac{1}{2}}}{n_1 p_1}}{\log n_1}$$
$$= e - 1/2 - \beta/\alpha. \tag{21}$$

Plugging the values of e from Table I we obtain Table III, from which we can draw three conclusions. Here, we focus on the jamming-dominant regime, since the noise-like regime is not particularly interesting.

First, the dependency of the power efficiency exponent e_η on the target network density (β) exhibits a bifurcated behavior: when $\beta - \gamma < \alpha/2 + 2/\alpha - 2$, e_η is an increasing function of β (the positive sign of β in Table III). The maximum e_η is obtained when $\beta - \gamma = \alpha/2 + 2/\alpha - 2$ and thus $e_\eta = 1/2 - \beta/\alpha$. In other words, *there is an optimal target network density for which the power efficiency of the network is maximized*. When $\beta - \gamma > \alpha/2 + 2/\alpha - 2$, e_η becomes a decreasing function of β (the negative sign of β in Table III). This is because the network consumes unnecessarily more power (larger n_1) than required for linear capacity scaling, leading to inefficiency.

Second, the dependency of the power efficiency exponent e_η on the jammer network density (γ) is relatively simpler: in the noise-like regime the dependency is none; and in the jamming-dominant regime, e_η is a decreasing function of γ.

Third, the maximum value, $e_\eta = 1/2 - \beta/\alpha$, is a decreasing function of β/α. In other words, small values of β and large values of α lead to higher power efficiency. This means that the network is more power-efficient when the network density is lower and the path loss exponent is higher, which corresponds to the condition of low interference.

VII. CONCLUSION

In this paper, we provide the scaling laws for the capacity of wireless networks in the presence of distributed jamming. We have shown the various capacity regimes delineated by the values of α, β and γ. We have also described the optimal communication schemes to achieve capacity bounds and the power efficiency of wireless networks. Our results can provide guidance for designing optimal wireless networking protocols that deal with large-scale distributed jamming.

APPENDIX A
THE PROOF OF THEOREM 1

Proof: First, we rescale the distances by $d_{1,0}$ and obtain

$$\hat{d}_{1,i,k} = d_{1,i,k}/d_{1,0}, \quad \hat{d}_{2,i,k} = d_{2,i,k}/d_{1,0},$$

$$\hat{H}_{1,i,k} = \frac{e^{j\theta_{i,k}}}{\hat{d}_{1,i,k}^{\alpha/2}}, \quad \hat{H}_{2,i,k} = \frac{e^{j\theta_{i,k}}}{\hat{d}_{2,i,k}^{\alpha/2}}.$$

The scaling changes the original network of area $2A^{1/2} \times A^{1/2}$ to that of area $2\sqrt{n_1} \times \sqrt{n_1}$, which in effect normalizes the network to the extended network. The cut-set capacity can be written as

$$C \leq \max_{\substack{E[tr(R_X)] \leq n_1 p_1 \\ E[tr(R_Z)] \leq n_2 p_2}} E[\log \det(I + d_{1,0}^{-\alpha} \hat{H}_2 R_Z \hat{H}_2^H$$
$$+ d_{1,0}^{-\alpha} \hat{H}_1 R_X \hat{H}_1^H) - \log \det(I + H_2 R_Z H_2^H)]. \tag{22}$$

We make the covariance matrices to be the identity matrices scaled by power levels, which reflects our assumptions of equal power allocation among the nodes and independent random phases in a fast fading channel [15].

Let A and B be two semi-positive definite matrices. Equation (11) has the following form:

$$\log \det A - \log \det B = \log \det AB^{-1}.$$

Using Hadamard's inequality,

$$\det AB^{-1} \leq \prod_{i=1}^{n} A_{i,i} \prod_{i=1}^{n} B_{i,i}^{-1}$$

TABLE III
SCALING EXPONENT OF POWER EFFICIENCY OF WIRELESS NETWORKS IN THE PRESENCE OF JAMMING

Noise-like regime $p_2 n_1^\gamma \le \infty$ as $n_1, n_2 \to \infty$	$\beta \ge \alpha/2 - 1$		$e_\eta = 1/2 - \beta/\alpha$	
	$\beta < \alpha/2 - 1$	$\beta > 0$	$e = \begin{cases} 3/2 - \alpha/2 + \beta(1-1/\alpha) & 2 \le \alpha < 3 \\ \beta(1/(\alpha-2) - 1/\alpha) & \alpha \ge 3 \end{cases}$	
		$\beta \le 0$	$e = \begin{cases} 3/2 - \alpha/2 + \beta(1-1/\alpha) & 2 \le \alpha < 3 \\ \beta(1 - 1/\alpha) & \alpha \ge 3 \end{cases}$	
Jammer-dominate regime $p_2 n_1^\gamma \to \infty$ as $n_1, n_2 \to \infty$	$\beta - \gamma \ge \alpha/2 + 2/\alpha - 2$		$e = 1/2 - \beta/\alpha$	
	$\beta - \gamma < \alpha/2 + 2/\alpha - 2$	$\beta - \gamma > 0$	$e = \begin{cases} 3/2 - \alpha/2 - \beta/\alpha + (\beta-\gamma)/(1-2/\alpha) & 2 \le \alpha < 3 \\ -\beta/\alpha + (\beta-\gamma)/(\alpha + 4/\alpha - 4) & \alpha \ge 3 \end{cases}$	
		$\beta - \gamma \le 0$	$e = \begin{cases} 3/2 - \alpha/2 - \beta/\alpha + (\beta-\gamma)/(1-2/\alpha) & 2 \le \alpha < 3 \\ -\beta/\alpha + (\beta-\gamma)/(1-2/\alpha) & \alpha \ge 3 \end{cases}$	

which is valid for any semi-positive definite matrix, and therefore valid for any linear combination of the form as appears within the log function in (11). We can rewrite (11) as

$$C \le \sum_i \log\left(1 + d_{1,0}^{-\alpha} p_2 (\hat{H}_2 \hat{H}_2^H)_{i,i} + d_{1,0}^{-\alpha} p_1 (\hat{H}_1 \hat{H}_1^H)_{i,i}\right)$$
$$- \log\left(1 + d_{1,0}^{-\alpha} p_2 (\hat{H}_2 \hat{H}_2^H)_{i,i}\right). \tag{23}$$

Now, we proceed to calculate the diagonal terms of the channel matrices. We fix the coordinate system such that the network is located in the area of $[0, n_1^{1/2}] \times [-n_1^{1/2}, n_1^{1/2}]$. Let $\hat{d}_1(x,y;x_i,y_i)(\hat{d}_2(x,y;x_i,y_i))$ denote the distance between a target (jammer) node located at (x,y) and a target (jammer) node located at (x_i, y_i); and c_i denote a certain constant (this convention is used in the remainder of the paper). In the scaled network, the density of target nodes is 1 (normalized to the extended network), and that of jammers is n_2/n_1. Using the fact that the nodes are uniformly randomly distributed, we can use integration in place of summation to calculate the diagonal terms of the squared channel matrices as follows

$$(\hat{H}_1 \hat{H}_1^H)_{i,i} = \sum_k |\hat{H}_{1,i,k}|^2 = \sum_k \hat{d}_{1,i,k}^{-\alpha}$$
$$= \int_0^{\sqrt{n_1}} \int_{-\sqrt{n_1}}^{-1} \hat{d}_1(x,y;x_i,y_i)^{-\alpha}\, dx dy$$
$$= c_1 x_i^{2-\alpha}, \tag{24}$$

$$(\hat{H}_2 \hat{H}_2^H)_{i,i} = \sum_k |\hat{H}_{2,i,k}|^2 = \sum_k \hat{d}_{2,i,k}^{-\alpha}$$
$$= \frac{n_2}{n_1} \int_0^{\sqrt{n_1}} \int_{-\sqrt{n_1}}^{\sqrt{n_1}} \hat{d}_2(x,y;x_i,y_i)^{-\alpha}\, dx dy$$
$$= c_2 \frac{n_2}{n_1} \left(\frac{d_{2,0}}{d_{1,0}}\right)^{2-\alpha}. \tag{25}$$

Note that when $\alpha = 2$, the integral is actually proportional to $\log n_1$. We ignore this detail because it does not affect the exponents in the scaling law. Also note that the integration range of x in (24) is from $-n_1^{1/2}$ to -1 because we are summing up the contributions from the target transmitters at

the left side of the cut to the ith target receiver at the right side of the cut. To avoid divergence in integration but without affecting the scaling behavior, we set the lower integration limit of x to 1 (recall that the average distance between the nearest target nodes in the rescaled network is 1). Similarly, the integration range of x in (25) is from $-n_1^{1/2}$ to $n_1^{1/2}$ because we are summing up jamming signals from both sides of the cut. Again, to avoid divergence we placed the restriction $min\, [d_2(x,y;x_i,y_i)] = d_{2,0}$ in (25). Plugging (24) and (25) into (23) and using the relationships in (4), we obtain

$$C \le \int_0^{\sqrt{n_1}} \int_1^{\sqrt{n_1}} [\log(1 + c_2 p_2 d_{2,0}^{-\alpha} + c_1 p_1 d_{1,0}^{-\alpha} x^{2-\alpha})$$
$$- \log(1 + c_2 p_2 d_{2,0}^{-\alpha})] dx dy. \tag{26}$$

Remark: The contribution from the jammers in (26), $c_2 p_2 d_{2,0}^{-\alpha}$, is on the same order of magnitude as the contribution from the nearest jammer to the target receiver alone, i.e., $c_2 p_2 d_{2,0}^{-\alpha}$. Using the definitions of β and γ we rewrite (26) as

$$C \le \int_0^{\sqrt{n_1}} \int_1^{\sqrt{n_1}} [\log(1 + c_2 p_2 n_1^\gamma + c_1 p_1 n_1^\beta x^{2-\alpha})$$
$$- \log(1 + c_2 p_2 n_1^\gamma)] dx dy. \tag{27}$$

∎

APPENDIX B
THE PROOF OF THEOREM 2

Before proceeding with the proof, we state the following elementary fact

$$\lim_{n_1 \to \infty} \int_{x_0}^{\sqrt{n_1}} x^{2-\alpha} dx = \begin{cases} n_1^{(3-\alpha)/2} & 2 \le \alpha < 3 \\ \log n_1 & \alpha = 3 \\ x_0^{3-\alpha}/(\alpha-3) & \alpha > 3 \end{cases} \tag{28}$$

where x_0 is a positive number.

We consider two major capacity regimes depending on whether $p_2 n_1^\gamma \le \infty$ or $p_2 n_1^\gamma \to \infty$ as $n_1, n_2 \to \infty$, which delineates the boundary between the noise-like regime and the jamming-dominant regime.

B.1. The noise-like regime: $p_2 n_1^\gamma \leq \infty$ as $n_1, n_2 \to \infty$

In this case, the contribution from jamming nodes in (27) is upper-bounded by a constant, say $c_3 - 1$, as n_1 and n_2 approaches infinity. In terms of scaling behavior, this case is equivalent to that where only noise is present. The scaling law results we obtained are identical to that in [15]. Below we look into different capacity regimes delineated by the values of β, which indicate the target node density and reflect the short-range SNR.

B.1.a The case of $\beta \geq \alpha/2 - 1$

Using Equation (27) and the fact $1 \leq x \leq n_1^{1/2}$ we obtain

$$C > \int_0^{\sqrt{n_1}} \int_1^{\sqrt{n_1}} [\log(c_3 + c_1 p_1 n_1^{\beta - \frac{\alpha}{2} + 1})] dx dy$$
$$C < \int_0^{\sqrt{n_1}} \int_1^{\sqrt{n_1}} [\log(c_3 + c_1 p_1 n_1^\beta)] dx dy$$

or

$$n_1 \log(c_3 + c_1 p_1 n_1^{\beta - \frac{\alpha}{2} + 1}) < C < n_1 \log(c_3 + c_1 p_1 n_1^\beta) \quad (29)$$

$$e = \lim_{n_1 \to \infty} \frac{\log C}{\log n_1} = 1 \quad (30)$$

B.1.b The case of $\beta < \alpha/2 - 1$

There are two sub-cases delineated by whether $\beta > 0$ or not.

B.1.b.i The case of $\beta > 0$

In this case, there exists x_0 in $[1, n_1^{1/2}]$ such that

$$c_1 p_1 n_1^\beta x_0^{2-\alpha} = c_3 \quad or \quad x_0 = \left(\frac{c_1 p_1 n_1^\beta}{c_3}\right)^{\frac{1}{\alpha-2}}. \quad (31)$$

We can break the integration in (27) into two parts delimited by x_0. In the second part, we use the fact $\log(1+x) < x$, which is tight for small values of x. Thus, we have

$$C > \int_0^{\sqrt{n_1}} \int_1^{x_0} [\log(c_3 + c_1 p_1 n_1^\beta x^{2-\alpha})] dx dy$$
$$+ \int_0^{\sqrt{n_1}} \int_{x_0}^{\sqrt{n_1}} [\log(c_3 + c_1 p_1 n_1^\beta x^{2-\alpha})] dx dy$$

$$= \begin{cases} c_4 \sqrt{n_1} x_0 \log n_1 + c_5 \sqrt{n_1} p_1 n_1^\beta n_1^{\frac{(3-\alpha)}{2}} & 2 \leq \alpha < 3 \\ c_4 \sqrt{n_1} x_0 \log n_1 + c_5 \sqrt{n_1} p_1 n_1^\beta \log n_1 & \alpha = 3 \\ c_4 \sqrt{n_1} x_0 \log n_1 + c_5 \sqrt{n_1} p_1 n_1^\beta x_0^{(3-\alpha)} & \alpha > 3 \end{cases}$$

$$e = \lim_{n_1 \to \infty} \frac{\log C}{\log n_1} = \begin{cases} 2 - \frac{\alpha}{2} + \beta & 2 \leq \alpha < 3 \\ \frac{1}{2} + \frac{\beta}{(\alpha-2)} & \alpha \geq 3 \end{cases} \quad (32)$$

B.1.b.ii The case of $\beta \leq 0$

Using $\log(1 + x) < x$ for small values of x, we have

$$C \leq \int_0^{\sqrt{n_1}} \int_1^{\sqrt{n_1}} [\log(c_3 + c_1 p_1 n_1^\beta x^{2-\alpha})] dx dy$$
$$\leq \int_0^{\sqrt{n_1}} \int_1^{\sqrt{n_1}} [(c_1 p_1 n_1^\beta x^{2-\alpha})/c_3] dx dy$$

$$= \begin{cases} c_6 \sqrt{n_1} p_1 n_1^\beta n_1^{\frac{(3-\alpha)}{2}} & 2 \leq \alpha < 3 \\ c_6 \sqrt{n_1} p_1 n_1^\beta \log n_1 & \alpha = 3 \\ c_6 \sqrt{n_1} p_1 n_1^\beta & \alpha > 3 \end{cases}$$

$$e = \lim_{n_1 \to \infty} \frac{\log C}{\log n_1} = \begin{cases} 2 - \frac{\alpha}{2} + \beta & 2 \leq \alpha < 3 \\ \frac{1}{2} + \beta & \alpha \geq 3 \end{cases} \quad (33)$$

B.2. The jamming-dominant regime: $p_2 n_1^\gamma \to \infty$ as $n_1, n_2 \to \infty$

In this case, the contribution from jamming signals is dominant and the noise can be ignored, using (7) we have

$$C \leq \int_0^{\sqrt{n_1}} \int_1^{\sqrt{n_1}} \left[\log(1 + \frac{c_1 n_1^{(\beta-\gamma)(1-2/\alpha)} x^{2-\alpha}}{c_2 \rho})\right] dx dy \quad (34)$$

Below we look into different capacity regimes delineated by the values of $\beta - \gamma$, which reflect the relative received signal levels of the target nodes and the jammers.

B.2.a The case of $\beta - \gamma > (\alpha/2 - 1)/(1 - 2/\alpha)$

Using Equation (27) and the fact $1 \leq x \leq n_1^{1/2}$ we obtain

$$C > \int_0^{\sqrt{n_1}} \int_1^{\sqrt{n_1}} \left[\log(1 + \frac{c_1 n_1^{(\beta-\gamma)/(1-2/\alpha) - \frac{\alpha}{2} + 1}}{c_2 \rho})\right] dx dy$$
$$C < \int_0^{\sqrt{n_1}} \int_1^{\sqrt{n_1}} \left[\log(1 + \frac{c_1 p_1 n_1^{(\beta-\gamma)/(1-2/\alpha)}}{c_2 \rho})\right] dx dy$$

or

$$C > n_1 \left(\log(1 + \frac{c_1 n_1^{(\beta-\gamma)/(1-2/\alpha) - \frac{\alpha}{2} + 1}}{c_2 \rho})\right)$$
$$C < n_1 \left(\log(1 + \frac{c_1 n_1^{(\beta-\gamma)/(1-2/\alpha)}}{c_2 \rho})\right)$$

$$e = \lim_{n_1 \to \infty} \frac{\log C}{\log n_1} = 1 \quad (35)$$

B.2.b The case of $\beta - \gamma < (\alpha/2 - 1)/(1 - 2/\alpha)$

Again, there are two sub-cases delineated by whether $\beta - \gamma > 0$ or not.

B.2.b.i The case of $\beta - \gamma > 0$

In this case, there exists x_0 in $[1, n_1^{1/2}]$ such that

$$\frac{c_1 n_1^{(\beta-\gamma)(1-2/\alpha)} x^{2-\alpha}}{c_2 \rho} = 1 \quad or \quad x_0 = \left(\frac{c_1 n_1^{(\beta-\gamma)(1-2/\alpha)}}{c_2 \rho}\right)^{\frac{1}{\alpha-2}} \quad (36)$$

We can break the integration (34) into two parts delimited by x_0. Using the (28) and the fact $\log(1+x) < x$, which is tight for small values of x, we have

$$C \leq \int_0^{\sqrt{n_1}} \int_1^{x_0} [\log(1 + \frac{c_1 n_1^{(\beta-\gamma)(1-2/\alpha)} x^{2-\alpha}}{c_2 \rho})] dx dy$$

$$+ \int_0^{\sqrt{n_1}} \int_{x_0}^{\sqrt{n_1}} [\log(1 + \frac{c_1 n_1^{(\beta-\gamma)(1-2/\alpha)} x^{2-\alpha}}{c_2 \rho})] dx dy$$

$$= \begin{cases} c_4 \sqrt{n_1} x_0 \log n_1 + c_5 \sqrt{n_1} p_1 n_1^{(\beta-\gamma)(1-2/\alpha)}. \\ n_1^{\frac{(3-\alpha)}{2}} & 2 \leq \alpha < 3 \\ c_4 \sqrt{n_1} x_0 \log n_1 + c_5 \sqrt{n_1} p_1 n_1^{(\beta-\gamma)(1-2/\alpha)}. \\ \log n_1 & \alpha = 3 \\ c_4 \sqrt{n_1} x_0 \log n_1 + c_5 \sqrt{n_1} p_1 n_1^{(\beta-\gamma)(1-2/\alpha)}. \\ x_0^{(3-\alpha)} & \alpha > 3 \end{cases}$$

$$e = \lim_{n_1 \to \infty} \frac{\log C}{\log n_1} = \begin{cases} 2 - \frac{\alpha}{2} + (\beta-\gamma)(1-2/\alpha) & 2 \leq \alpha < 3 \\ \frac{1}{2} + (\beta-\gamma)(\alpha+4/\alpha-4) & \alpha \geq 3 \end{cases} \tag{37}$$

$B.2.b.ii$ The case of $\beta - \gamma \leq 0$

Using Equation (34) and the fact $\log(1+x) < x$, which is tight for small values of x we have

$$C \leq \int_0^{\sqrt{n_1}} \int_1^{\sqrt{n_1}} [\log(1 + \frac{c_1 n_1^{(\beta-\gamma)(1-2/\alpha)} x^{2-\alpha}}{c_2 \rho})] dx dy$$

$$\leq \sqrt{n_1} \int_1^{\sqrt{n_1}} (\frac{c_1 n_1^{(\beta-\gamma)(1-2/\alpha)} x^{2-\alpha}}{c_2 \rho}) dx$$

$$= \begin{cases} c_6 \sqrt{n_1} n_1^{(\beta-\gamma)(1-2/\alpha)} n_1^{\frac{(3-\alpha)}{2}}/\rho & 2 \leq \alpha < 3 \\ c_6 \sqrt{n_1} n_1^{(\beta-\gamma)(1-2/\alpha)} \log n_1/\rho & \alpha = 3 \\ c_6 \sqrt{n_1} n_1^{(\beta-\gamma)(1-2/\alpha)}/\rho & \alpha > 3 \end{cases}$$

$$e = \lim_{n_1 \to \infty} \frac{\log C}{\log n_1} = \begin{cases} 2 - \frac{\alpha}{2} + (\beta-\gamma)(1-2/\alpha) & 2 \leq \alpha < 3 \\ \frac{1}{2} + (\beta-\gamma)(1-2/\alpha) & \alpha \geq 3 \end{cases} \tag{38}$$

REFERENCES

[1] System Description and Operating Principles for High Throughput Enhancements to 802.11, IEEE 802.11-04/0870r, 2004.

[2] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 42, pp. 388–404, Mar. 2000.

[3] H. Huang, N. Ahmed and P. Karthik, "On a new type of denial of service attack in wireless networks: the distributed jammer network," *IEEE Transactions on Wireless Communications*, Vol. 10, No.7, pp. 2316–2324, 2011.

[4] M. Balakrishnan, H. Huang, R. Asorey-Cacheda, S. Misra, S. Pawar, and Y. Jaradat, "Null frequency jamming of routing protocols in wireless ad hoc networks," *IEEE Transactions on Wireless Communications*, Vol.11, No. 11, pp. 3860–3868, 2012.

[5] A. Goldsmith, *Wireless Communications*, Cambridge University Press, 2005.

[6] A. Jovicic, P. Viswanath, and S. R. Kulkarni, "Upper bounds to transport capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 50, no. 11, pp. 2555–2565, Nov. 2004.

[7] L.L. Xie and P.R. Kumar, "A network information theory for wireless communications: Scaling laws and optimal operation," *IEEE Trans. Inf. Theory*, vol. 50, no. 5, pp. 748–767, May 2004.

[8] O. Lévêque and E. Telatar, "Information theoretic upper bounds on the capacity of large, extended ad hoc wireless networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 3, pp. 858–865, Mar. 2005.

[9] F. Xue, L.L. Xie, P.R. Kumar, "The transport capacity of wireless networks over fading channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 3, pp. 834–847, Mar. 2005.

[10] L.L. Xie and P.R. Kumar, "On the path-loss attenuation regime for positive cost and linear scaling of transport capacity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 52, no. 6, pp. 2313–2328, Jun. 2006.

[11] A. Özgür, O. Lévêque, and E. Preissmann, "Scaling laws for one and two-dimensional random wireless networks in the low attenuation regime," *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3573–3585, Oct. 2006.

[12] S. Aeron and V. Saligrama, "Wireless ad hoc networks: Strategies and scaling laws for the fixed SNR regime," *IEEE Trans. Inf. Theory*, vol. 53, no. 6, pp. 2044–2059, Jun. 2007.

[13] M. Franceschetti, O. Dousse, D. Tse, and P. Thiran, "Closing the gap in the capacity of wireless networks via percolation theory," *IEEE Trans. Inf. Theory*, vol. 53, no. 3, pp. 1009–1018, Mar. 2007.

[14] A. Özgür, O. Lévêque, and D. Tse, "Hierarchical cooperation achieves optimal capacity scaling in ad-hoc networks," *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3549–3572, Oct. 2007.

[15] A. Özgür, R. Johari, D. Tse, and O. Lévêque, "Information-theoretic operating regimes of large wireless networks," *IEEE Trans. Inf. Theory*, vol. 56, no. 1, pp. 427–437, Jan. 2010.

[16] C. Schleher, "Electronic Warfare in the Information Age," MArtech House, 1999.

[17] A.D. Wood and J. Stankovic, "Denial of service in sensor networks," *IEEE Computer*, vol. 35, no. 10, pp. 54–62, 2002.

[18] R. Mallik, R. Scholtz, and G. Papavassilopoulos, "Analysis of an on-off jamming situation as a dynamic game," *IEEE Trans. Commun.*, vol. 48, no. 8, pp. 1360–1373, Aug. 2000.

[19] R. Negi and A. Perrig, "Jamming analysis of MAC protocols," *Carnegie Mellon Technical Memo*, 2003.

[20] J. Bellardo and S. Savage, "802.11 denial-of-service attacks: Real vulnerabilities and practical solutions," in *Proc. of the USENIX Security Symposium*, pp. 15–28, 2003.

[21] A. Wood, J. Stankovic, and S. Son. "JAM: A jammed-area mapping service for sensor networks," in *Proc. of IEEE Real-Time Systems Symposium*, pp. 286–297, 2003.

[22] G. Noubir and G. Lin, "Low-power DoS attacks in data wireless LANs and countermeasures," *SIGMOBILE Mob. Comput. Commun. Rev.*, vol. 7, No. 3, pp. 29–30, 2003.

[23] J. M. McCune, E. Shi, A. Perrig and M. K. Reiter, "Detection of denial-of-message attacks on sensor network broadcasts," in *Proc. IEEE Symposium on Security and Privacy*, 2005.

[24] W. Xu et al., "The Feasibility of Launching and Detecting Jamming Attacks in Wireless Networks," in *Proc. of ACM Int'l. Symp. Mobile Ad Hoc Net. and Comp.*, pp. 46–57, 2005.

[25] W. Xu, T. Wood, W. Trappe, and Y. Zhang, "Channel surfing and spatial retreats: defenses against wireless denial of service," in *Proc. of ACM workshop on Wireless Security*, pp. 80–89, 2004.

[26] M. Li, I. Koutsopoulos, and R. Poovendran, "Optimal jamming attacks and network defense policies in wireless sensor networks," in *Proc. of IEEE Infocom*, pp. 1307–1315, May 2007.

[27] A. D. Wood, J. Stankovic, and G. Zhou, "DEEJAM: defeating energy-efficient jamming in IEEE 802.15.4-based wireless networks," in *Proc. of ACM SECON*, pp. 60–69, 2007.

[28] J. T. Chiang and Y.-C. Hu, "Cross-layer jamming detection and mitigation in wireless broadcast networks," in *Proc. of ACM MobiCom*, pp. 346–349, New York, NY, USA, 2007.

[29] X. Liu, R. Rajaraman, G. Noubir, B. Thapa, C. King and E. Bayrak-taroglu, "On the Performance of IEEE 802.11 under Jamming," in *Proc. of IEEE Infocom*, Apr. 2008.

[30] P. Tague, D. Slater, G. Noubir, and R. Poovendran. "Linear Programming Models for Jamming Attacks on Network Traffic Flows," in *Proc. of WiOpt*, 2008.

[31] C. E. Shannon, "Communication theory of secrecy systems," *The Bell System Technical Journal*, vol. 28, pp. 656–715, 1949.

[32] A. Wyner, "The wire-tap channel," *The Bell System Technical Journal*, vol. 54, no. 8, pp. 1355–1387, Oct. 1975.

[33] I. Csiszar and J. Korner, "Broadcast channels with confidential messages," *IEEE Transactions on Information Theory*, vol. 24, no. 3, pp. 339–348, May 1978.

[34] S. Leung-Yan-Cheong and M. Hellman, "The Gaussian wire-tap channel," *IEEE Transactions on Information Theory*, vol. 24, no. 4, pp. 451–456, Jul. 1978.

[35] M. Haenggi, "The secrecy graph and some of its properties," in *Proc. of 2008 IEEE International Symposium on Information Theory (ISIT)*, pp. 539–543, Jul. 2008.

[36] P. C. Pinto, J. Barros, and M. Win, "Physical-layer security in stochastic wireless networks," in *Proc. of 11th IEEE International Conference on Communication Systems (ICCS)*, pp. 974–979, Nov. 2008.

[37] P. C. Pinto, J. Barros, and M. Win, "Wireless physical-layer security: The case of colluding eavesdroppers," in *Proc. 2009 IEEE International Symposium on Information Theory (ISIT)*, pp. 2442–2446, Jun. 2009.

[38] P. C. Pinto, J. Barros, and M. Win, "Secure communication in stochastic wireless networks-Part I: Connectivity," *IEEE Transactions on Information Forensics Security*, vol. 7, no. 1, pp. 125–138, 2012.

[39] H. Wang, X. Zhou, and M. C. Reed, "Physical layer security in cellular networks: A stochastic geometry approach," *IEEE Transactions on Wireless Communications*, vol.12, no.6, pp.2776–2787, 2013.

[40] X. Zhou, R. K. Ganti, J. G. Andrews, and A. Hjorungnes, "On the throughput cost of physical layer security in decentralized wireless networks," *IEEE Transactions on Wireless Communications*., vol. 10, no. 8, pp. 2764–2775, 2011.

[41] P. C. Pinto, J. Barros, and M. Win, "Secure communication in stochastic wireless networks-Part II: Maximum rate and collusion," *IEEE Transactions on Information Forensics Security*, vol. 7, no. 1, pp. 139–147, 2012.

[42] O. O. Koyluoglu, C. E. Koksal, and H. E. Gamal, "On secrecy capacity scaling in wireless networks," *IEEE Transaction on Information Theory*, vol. 58, no. 5, pp. 3000–3015, 2012.

[43] G. Geraci, S. Singh, J. G. Andrews, J. Yuan, and I. B. Collings, "Secrecy rates in the broadcast channel with confidential messages and external eavesdroppers," *IEEE Transactions on Wireless Communications*, vol. 13, no. 5, pp. 2931–2943, 2014.

Amjad Abu-Baker received the B.S. degree from the Jordan University of Science and Technology, Jordan, in 2000, and the M.S. and the Ph.D. degrees in Electrical Engineering from the New Mexico State University, USA, in 2009 and 2012 respectively. Currently, he is an assistant professor with the Department of Telecommunication Engineering at the Yarmouk University, Jordan. His research interests include wireless communications and networking, wireless sensor networks, energy harvesting wireless sensor networks, information theory, and coding theory.

Rafael Asorey-Cacheda received the M.Sc. degree in Telecommunication Engineering (major in Telematics and Best Master Thesis Award) and the Ph.D. Degree (cum laude and Best PhD Thesis Award) in Telecommunication Engineering from the University of Vigo, Spain, in 2006 and 2009 respectively. He was a researcher with the Information Technologies Group, University of Vigo, Spain until 2009. Between 2008 and 2009 he was also R&D Manager at Optare Solutions, a Spanish telecommunications company. Between 2009 and 2012 held an Ángeles Alvariño position, Xunta de Galicia, Spain. Currently, he is a professor at the Defense University Center, University of Vigo. His interests include content distribution, high-performance switching, video transcoding, peer-to-peer networking and wireless networks.

Hong Huang received his B.E. degree from Tsinghua University, Beijing, China, and M.S. and Ph.D. degrees from Georgia Institute of Technology in 2000 and 2002, respectively, all in electrical engineering. He is currently an associate professor with the Klipsch School of Electrical and Computer Engineering at the New Mexico State University. His current research interests include wireless sensor networks, mobile ad hoc networks, network security, and optical networks. He is a member of IEEE.

Satyajayant Misra (SM'5, M'09) is an associate professor in computer science at New Mexico State University. His research interests include wireless networks and the Internet, supercomputing, and smart grid architectures and protocols. He has been involved in several IEEE journal editorial boards and conference executive committees (Communications on Surveys and Tutorials, Wireless Communications Magazine, MOBICOM 2015, ANTS 2014, SECON 2010, INFOCOM 2012). He has authored more than 40 peer-reviewed IEEE/ACM journal articles and conference proceedings. One of his co-authored papers was a runner-up to the best-paper award at IEEE ICNP 2010.

Yousef Jaradat received the B.S. in electrical and computer engineering from Jordan University of Science and Technology, Irbid, Jordan, in 2000, and the M.S. in computer science from Amman Arabia University, Amman, Jordan, in 2007, and the Ph.D. in electrical and computer engineering from New Mexico State University, Las Cruces, New Mexico, USA, in 2012. He is currently an assistant professor with the Department of Computer and Communication Engineering at Al-Zaytoonah University of Jordan. His research includes work in the capacity of wireless networks, Obstacles analysis in the network area, and simulation of wireless networks.

Reza Tourani received his B.S. in computer engineering from IAUT, Tehran, Iran, in 2008, and M.S. in computer science from New Mexico State University, Las Cruces, NM, USA, in 2012. From 2013, he started his Ph. D. at New Mexico State University. His research interests include cyber-physical system architecture and security protocols, cache optimization, future Internet architecture, and security and privacy in information-centric networking.

Mohammad Masoud is an assistant professor in computer and communication engineering at Al-zaytoonah University of Jordan. His research includes work in the design and measurement of computer network and their applications, P2P networks, routing protocols and ad hoc networks.

Ismael Jannoud is an Associate professor in computer and communication engineering at Al-zaytoonah University of Jordan, Jordan, and Damascus University, Syria. His research includes work in the digital image processing, computer vision, pattern recognition, multimedia processing, and computer networks.