Sparse Control and Compressed Sensing in Networked Switched Systems

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Abstract

In this paper, we study the problems of sparse control and data transmission for network switched control systems. First, the networked control system is modeled as switched control systems. The new stability criterion and stabilization criterion are presented based on a sparse control design methodology and the Lyapunov stability theory. Then, according to the structure of the systems, data transmission through the unreliable network from sensors to the controller becomes unavoidable. In the transmission, real-time compressed sensing (CS) is effectively employed, which can reduce the size of transfer data and increase the reliability of data transmission. The controller design method and the CS approach are very effective, which has been explained and verified via simulation studies.

Index Terms

Sparse control, compressed sensing (CS), packetized predictive control (PPC), networked control systems (NCS), switched systems.

I. Introduction

As is well known, controllers, plants and sensors are often connected over a network medium, which comprise networked control systems. However, owing to the limits of the capacities

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and bandwidth of the physical communication network, network-induced delays, and packet drops, the network control systems suffer from unreliable communications. Thus, packet loss is considered in building the networked control system model. The reason for employing networked control systems comes from their great advantages of resources sharing, low cost, simple system maintenance, etc. It also has wide applications, for example, robots, unmanned aerial vehicles, aircrafts, etc [1], [2]. With development of the modern computer technology, much attention has been paid to the study of stability analysis and control design of networked control systems from researchers in the system and control community due to the advantages and wide applications of networked control systems, and a great number of significant results concerned with networked control systems have been reported [3], [4], [5], [6], [7], [8], [9].

On the other hand, reducing the size of transfer data in networked control system is also an interesting research topic. Compressed sensing (CS) is an active field that has attracted considerable research interests in the signal processing community [10], [11] since the important works of Candes, et al. [12] and Donoho [13]. And it is widely used in signal transmission, compression, and recovery. The Shannon/Nyquist sampling theorem specifies that to avoid losing information when capturing a signal, one must sample at least two times faster than the signal bandwidth. In many applications, including digital image and video cameras, the Nyquist rate is too high. We can capture the signals at a rate far below the Nyquist rate by using CS.

In networked control systems, communications between controllers and plants, also between sensors and controllers, are made through unreliable and rate-limited communication links such as wireless networks and the Internet; thus, considerations of both control and communication aspects become necessary. In particular, packetized predictive control (PPC) has been shown to have favorable stability and performance properties, especially in the presence of packet-drop [14], [15], [16], [17], [18]. In PPC, the controller output is obtained through minimizing a finite-horizon cost function on-line and in a receding horizon manner. Each control packet contains a sequence of tentative plant inputs for a finite horizon of future time instants and is transmitted through a communication channel. Successfully received packets are stored in the buffer to prepare to be used if the later packets are dropped. In [19], Nagahara, et al. have shown that the sparse packetized predictive control performs well for deterministic systems. But many practical network systems are subject to random abrupt changes in their inputs, internal variables and other system parameters, which can not only be represented by linear time-invariant systems. Thus, for accurate description of networked systems, networked switched systems are presented and investigated in the literature [20], [21]. The effectiveness is questionable, when networked control systems are modelled as switched systems [20], [21]. Also, in networked control systems, sensors are often far away from the controller, and data from the sensors still needs to be transferred. If large size of data needs to be transferred, and the network bandwidth is not big enough, time delay and uncertainty would happen. Thus, for reducing the effectiveness of unreliable and rate-limited communication links, we use CS method to reduce the transferred data size in the feedback channel.

The contributions of the paper are two-folds. First, we extend the sparse packetized predictive control method from linear time-invariant system to linear switched system in bit-rate limited networks. Second, we introduce the compressed sensing method to the information communication in control system. At first, the switched control model is introduced. Then, based on the result in [19], we present the sparse PPC method for networked control switched systems. Then, motivated by the result in [22], sequential CS method is used to solve the data transmission in the feedback channel of control systems. Finally, illustrative examples are presented to demonstrate the effectiveness of the developed controller design method and the CS method.

The paper is organized as follows. In section II, the networked control problem and the data transmission problem in feedback channel are formulated, and the switched control system model and the structure of systems are presented. In Section III, the sparse PPC controller is proven to guarantee the practical stability of the system. In Section IV, We employ CS method to solve the data transmission problem of networked control systems. Illustrative examples are given to demonstrate the effectiveness of proposed methods. in Section V, and finally the conclusions are presented in Section VI.

Notations: $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ represent the set of real n-vector and $m\times n$ matrices, respectively. $\mathbb{N}_0$ denotes $\left\{ \begin{array}{cccc} 1 & 2 & 3 & ... \end{array} \right\}$. The superscript "T" stands for matrix transpose. The notation $P>0\,(\geq 0)$ means that the matrix P is positive (semi)definite. I_n denotes an identity matrix with dimension n and $0_{m,n}$ denotes an $m\times n$ dimension zero matrix. The symbol " $*$ " is used to denote the symmetric terms in a block matrix P, $\{P\}_i$ represents the i^{th} row of its explicitly expressed block structure. Also define $\|N\|_1 \triangleq |N_1| + ... + |N_n|$, $\|N\|_2 \triangleq \sqrt{N^TN}$, and $\|N\|_P \triangleq \sqrt{N^TPN}$, where $N = \left[\begin{array}{cccc} N_1 & N_2 & \cdots & N_n \end{array} \right]$.

II. PROBLEM FORMULATION AND MODEL REFORMULATION

In this section, the model of switched system is presented, then some relationships between controller signals and sensor signals are introduced which will be used in the subsequent development. We consider the following discrete-time linear plant model:

$$x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k), \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}$ is the control signal, $\sigma(k)$ is a piecewise constant function of time, called switching signal which takes its values in the finite set $\mathcal{N} = \left\{ \begin{array}{cccc} 1 & 2 & \dots & \bar{N} \end{array} \right\}$. As in [23], we assume that the switching signal $\sigma(k)$ is unknown, but its instantaneous value is available in real time. A_i and B_i are known real constant matrices of appropriate dimensions representing the nominal systems for each $\sigma(k) \in \mathcal{N}$. We assume that the realization (A_i, B_i) is reachable.

In this paper, we are interested in a networked control architecture where the controller communicates with the plant actuator through an unreliable channel, which can be seen in Fig. 1. The forward channel denotes the signal channel including discrete controller, unreliable network, buffer, and control plant in Fig. 1. The feedback channel denotes the signal channel containing sensor, unreliable network, and discrete controller. We model the forward channel packet-drops as follows:

$$d(k) \triangleq \begin{cases} 1, & \text{if packet-drop occurs at instant } k, \\ 0, & \text{if packet-drop does not occur at instant } k. \end{cases}$$

At each time instant k, the controller uses the state $x(k)$ of the system in (1) to calculate and send a control packet of the form

$$U(x(k)) \triangleq \left[\begin{array}{cccc} u_0(x(k)) & u_1(x(k)) & \dots & u_{N-1}(x(k)) \end{array} \right]^T \in \mathbb{R}^N, \tag{2}$$

to the plant input node. As in [16], in order to make the system robust against packet drops, buffering is needed. Assume that at time instant k, we have $d(k) = 0$, then the data packet $U(x(k))$ defined in (2) is successfully received at the plant input side. Then, we store this packet in the buffer. If the next packet $U(x(k+1))$ is dropped, then the plant input $u(k+1)$ is set to $u_1(x(k))$, the second element of $U(x(k))$. The elements of $U(x(k))$ are then successively used until the packet is received. The sequence of buffer states $b(k)$ satisfies that

$$b(k) = d(k)Sb(k-1) + (1 - d(k))U(x(k)), \tag{3}$$

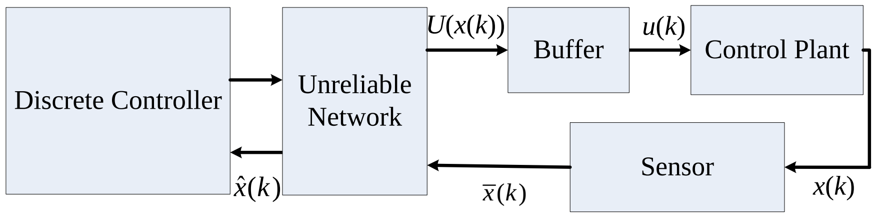

Fig. 1. The Structure of networked switched systems

where $b(0) = 0 \in \mathbb{R}^N$ and with

$$S \triangleq \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 1 & 0 \\ 0 & \dots & \dots & 0 & 1 \\ 0 & \dots & \dots & \dots & 0 \end{bmatrix} \in \mathbb{R}^{N\times N}.$$

The buffer states give rise to the plant inputs in (3) via

$$u(k) = [\, 1 \quad 0 \quad \dots \quad 0 \,] b(k). \tag{4}$$

This paper is concerned with the problems of sparse controller design for networked switched system and real-time CS data transmission in feedback channel. The purpose is to present the effective controller design method and real-time CS method to solve the stability and data transmission problems of networked switched systems. Using the above definition, the problems to be studied are formulated as follows:

1) *Stability Analysis and Sparse Control* – we investigate under what conditions, the networked switched control system is stable with a designed sparse controller.

2) *Compressed Sensing in Feedback Channel* – we introduce the concept of CS method into control systems and also keep the data in the real time.

III. Sparse Packetized Predictive Control for Networked Switched Systems

In [19], Nagahara, et al. has presented the good result of sparse packetized predictive control. But they only discuss the switched control concerned about packet loss, and do not consider about abrupt changes of the internal variables of the system. In this section, we generalize the idea in [19] to the networked switched systems and also consider abrupt changes of the internal variables of the systems. First, we propose to use the dynamic l_1/l_2 optimization. The controller optimization cost function is as follows:

$$J(U,x(k)) \triangleq \|x(N|k)\|_{P_{\sigma(k)}}^2 + \sum_{i=0}^{N-1} \|x(i|k)\|_Q^2 + \mu \sum_{i=0}^{N-1} |u_i| \,, \tag{5}$$

where $P_{\sigma(k)} > 0$, $Q > 0$, $\mu > 0$, $i \in \left\{ \begin{array}{cccc} 0 & 1 & \ldots & N \end{array} \right\}$ in $x(i|k)$, and $x(i|k)$ is predicted plant states, which are calculated by (1).

Now we introduce the matrices:

$$\Gamma \triangleq \begin{bmatrix} B_{\sigma(k)} & 0 & \ldots & 0 \\ A_{\sigma(k+1)}B_{\sigma(k)} & B_{\sigma(k+1)} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \prod_{i=1}^{N-1} A_{\sigma(k+i)}B_{\sigma(k)} & \prod_{i=1}^{N-2} A_{\sigma(k+i+1)}B_{\sigma(k+1)} & \ldots & B_{\sigma(k+N-1)} \end{bmatrix}, \tag{6}$$

$$\Upsilon \triangleq \begin{bmatrix} A_{\sigma(k)} \\ \prod_{i=0}^{1} A_{\sigma(k+i)} \\ \vdots \\ \prod_{i=0}^{N-1} A_{\sigma(k+i)} \end{bmatrix}, \ \bar{Q}_{\sigma(k)} = \text{blockdiag}\left\{ \begin{array}{cccc} Q & \ldots & Q & P_{\sigma(k)} \end{array} \right\},$$

according to the definition of the above matrices in (6), we can rewrite the optimization cost function as follows:

$$J(U,x(k)) = \left\|G_{\sigma(k)}U - H_{\sigma(k)}x(k)\right\|_2^2 + \mu \|U\|_1 + \|x(k)\|_Q^2 \,,$$

where $G_{\sigma(k)} \triangleq \bar{Q}_{\sigma(k)}^{1/2}\Gamma$, $H_{\sigma(k)} \triangleq -\bar{Q}_{\sigma(k)}^{1/2}\Upsilon$.

Similar to [16], we analyze practical stability of l_1/l_2 PPC with bounded packet drops for the networked switched system. For this purpose, the value function is analyzed as follows:

$$V(x) \triangleq \min_U J(U,x). \tag{7}$$

Then the following theorems and lemmas are presented.

Theorem 1: Let $r > 0$, $\tilde{Q} > 0$, i, $j \in \mathcal{N}$ and $K_i = \bar{K}_i\tilde{P}_i$ is the closed-loop state-feedback control gain, then $\tilde{P}_i > 0$ is the solution for the following Linear Matrix Inequality (LMI):

$$\begin{bmatrix} -\tilde{P}_i + \tilde{Q}_i & \bar{K}_i^T B_i^T + \tilde{P} A_i^T & \bar{K}_i^T \\ * & -\tilde{P}_j & 0 \\ * & * & -\frac{1}{r} \end{bmatrix} < 0, \tag{8}$$

where $Q = \tilde{P}_i^{-1}\tilde{Q}_i\tilde{P}_i^{-1}$, $P_i = \tilde{P}_i^{-1} > 0$, and $u(k) = K_{\sigma(k)}x(k)$.

Proof: Construct the Lyapunov function as $\bar{V}(x) = x^T(k)P_{\sigma(k)}x(k)$. Then we obtain

$$\Delta\bar{V}(x) = x^T(k+1)P_{\sigma(k+1)}x(k+1) - x^T(k)P_{\sigma(k)}x(k).$$

According to the system in (1) and the inequality in (8), we have

$$\begin{aligned}\Delta\bar{V}(x) &= x^T(k)\left[\left(A_{\sigma(k)} + B_{\sigma(k)}K_{\sigma(k)}\right)^T P_{\sigma(k+1)}\left(A_{\sigma(k)} + B_{\sigma(k)}K_{\sigma(k)}\right) - P_{\sigma(k)}\right]x(k)\\ &\leq -x^T(k)\left[Q + rK_{\sigma(l)}^T K_{\sigma(l)}\right]x(k) \leq 0.\end{aligned}$$

According to the Lyapunov stability theory, the system with the controller is stable. □

Lemma 1 [16]: For any $x \in \mathbb{R}^n$, we have

$$\lambda_{\min}(Q)\left\|x\right\|_2^2 \leq V(x) \leq \phi(\left\|x\right\|_2),$$

where

$$\begin{aligned}\phi(\left\|x\right\|_2) &\triangleq a_1\left\|x\right\|_2 + (a_2 + \lambda_{\max}(Q))\left\|x\right\|_2^2,\ G_{\sigma(k)}^+ \triangleq \left(G_{\sigma(k)}^T G_{\sigma(k)}\right)^{-1} G_{\sigma(k)}^T,\\ a_1 &\triangleq \max_{\sigma(k)\in\mathcal{N}} \mu\sqrt{n}\lambda_{\max}\left(G_{\sigma(k)}^+ H_{\sigma(k)}\right),\ a_2 \triangleq \max_{\sigma(k)\in\mathcal{N}} \lambda_{\max}^2\left[\left(G_{\sigma(k)}G_{\sigma(k)}^+ - I\right)H_{\sigma(k)}\right].\end{aligned}$$

Having established the above results, we introduce the iterated mapping f^i with implicit (open-loop optimal) input

$$U(x) = \begin{bmatrix} u_0(x) & \ldots & u_{N-1}(x)\end{bmatrix}^T = \arg\min_U J(U,x),$$

by

$$f^i(x) \triangleq \prod_{j=0}^{i-1} A_{\sigma(j)}x + \sum_{l=0}^{i-1}\prod_{j=i-l-1}^{i-1} A_{\sigma(j)}B_{\sigma(l)}u_l(x),\ i = 1, 2, .., N.$$

This mapping describes the plant state evolution during periods of consecutive packet drops.

Assumption 1: (Packet-drop). The number of consecutive packet-drop is uniformly bounded by the prediction horizon minus one, i.e., we have $m_i \leq N-1, \forall i \in \mathbb{N}_0$.

In practice, the likelihood of this many consecutive packet drops is low but still possible. If this rare event occurs, the designed controller can not guarantee the stability of the networked system, but then the fault diagnosis of the system would activate. The issues related to fault diagnosis is out of the scope of this paper.

Theorem 2: (Practical stability) Assume that $Q > 0,\ P_j > 0,\ j \in \mathcal{N}$ satisfies (8), with

$$r = \frac{\mu^2}{4\epsilon},\ \epsilon > 0.$$

Then for any $x \in \mathbb{R}^n$, we have

$$\|x(k)\|_2 \leq \rho^{\frac{i+1}{2}} \sqrt{\frac{\phi(\|x(k_0)\|_2)}{\lambda_{\min}(Q)}} + \Lambda,$$

where $i \in \mathbb{N}_0$ is such that $k \in \{k_i + 1, ..., k_{i+1}\}$,

$$\begin{aligned} \Lambda &\triangleq \sqrt{\frac{1}{1-\rho}\left(\frac{\epsilon}{\lambda_{\min}(Q)} + \varepsilon\right)}, \\ \rho &\triangleq 1 - \frac{\lambda_{\min}(Q)}{a_1 + a_2 + \lambda_{\max}(Q)}, \\ \varepsilon &> 0. \end{aligned}$$

Proof: Set $i \in \{1, ..., N-1\}$ and consider the sequence

$$\tilde{U} = \left\{ \begin{array}{ccccccc} u_i(x) & u_{i+1}(x) & ... & u_{N-1}(x) & \tilde{u}_N & ... & \tilde{u}_{N+i-1} \end{array} \right\},$$

where $\tilde{u}_{N+j} (j = 0, 1, ..., i-1)$ is given by

$$\tilde{u}_{N+j} = K_{\sigma(N+j)} \tilde{x}_{N+j}, \tilde{x}_{N+j+1} = A_{\sigma(N+j)} \tilde{x}_{N+j} + B_{\sigma(N+j)} \tilde{u}_{N+j},$$

with $K_{\sigma(N+j)}$ in Theorem 1 and where $\tilde{x}_N = f^N(x)$. Then we can obtain

$$\begin{aligned} J(\tilde{U}, f^i(x)) &= \|\tilde{x}_{N+i}\|^2_{P_{\sigma(N+i)}} + \sum_{l=i}^{N-1} \left\{ \left\|f^l(x)\right\|^2_Q + \mu \left|u_l(x)\right| \right\} + \sum_{l=N}^{N+i-1} \left\{ \|\tilde{x}_l\|^2_Q + \mu \left|\tilde{u}_l\right| \right\} \quad (9) \\ &= V(x) - \sum_{l=0}^{i-1} \left\{ \left\|f^l(x)\right\|^2_Q + \mu \left|u_l(x)\right| \right\} \\ &\quad + \|\tilde{x}_{N+i}\|^2_{P_{\sigma(N+i)}} - \left\|f^N(x)\right\|^2_{P_{\sigma(N)}} + \sum_{l=N}^{N+i-1} \left\{ \|\tilde{x}_l\|^2_Q + \mu \left|\tilde{u}_l\right| \right\} \\ &= V(x) - \sum_{l=0}^{i-1} \left\{ \left\|f^l(x)\right\|^2_Q + \mu \left|u_l(x)\right| \right\} \\ &\quad + \sum_{l=N}^{N+i-1} \left\{ \|\tilde{x}_{l+1}\|^2_{P_{\sigma(l+1)}} - \|\tilde{x}_l\|^2_{P_{\sigma(l)}} + \|\tilde{x}_l\|^2_Q + \mu \left|\tilde{u}_l\right| \right\}. \end{aligned}$$

According to the closed-loop systems form:

$$\begin{aligned} \tilde{x}_{l+1} &= \left(A_{\sigma(l)} + B_{\sigma(l)} K_{\sigma(l)}\right) \tilde{x}_l, \\ \tilde{u}_l &= K_{\sigma(l)} \tilde{x}_l, \ l = N, N+1, ..., N+i-1, \end{aligned}$$

where $K_{\sigma(l)}$ can be obtained by Theorem 1. The last sum term in (9) has the upper bound:

$$\begin{aligned} & \|\tilde{x}_{l+1}\|^2_{P_{\sigma(l+1)}} - \|\tilde{x}_l\|^2_{P_{\sigma(l)}} + \|\tilde{x}_l\|^2_Q + \mu\,|\tilde{u}_l| \\ = \;& \tilde{x}_l^T \Psi_{\sigma(l)} \tilde{x}_l - \frac{\mu^2 N}{4\epsilon}\left(\left|K_{\sigma(l)}\right| - \frac{2\epsilon}{\mu N}\right)^2 + \frac{\epsilon}{N}, \end{aligned}$$

where

$$\Psi_{\sigma(l)} = \left(A_{\sigma(l)} + B_{\sigma(l)}K_{\sigma(l)}\right)^T P_{\sigma(l+1)}\left(A_{\sigma(l)} + B_{\sigma(l)}K_{\sigma(l)}\right) - P_{\sigma(l)} + Q + \frac{\mu^2 N}{4\epsilon} K^T_{\sigma(l)} K_{\sigma(l)}.$$

According to Theorem 1, the inequality in (8) is equivalent to $\Psi_{\sigma(l)} < 0$. Then we have

$$\begin{aligned} J(\tilde{U}, f^i(x)) &\leq V(x) - \sum_{l=0}^{i-1}\left\{\left\|f^l(x)\right\|^2_Q + \mu\,|u_l(x)|\right\} + \epsilon \\ &\leq V(x) - \lambda_{\min}(Q)\,\|x\|_2^2 + \epsilon, \end{aligned}$$

according the fact that $f^0(x) = x$ and the inequality in (7), we have

$$V(f^i(x)) \leq J(\tilde{U}, f^i(x)) \leq V(x) - \lambda_{\min}(Q)\,\|x\|_2^2 + \epsilon. \tag{10}$$

For the case $i = N$, we consider the sequence $\tilde{U} = \left\{ \begin{array}{cccc} \tilde{u}_N & \tilde{u}_{N+1} & \ldots & \tilde{u}_{2N-1} \end{array} \right\}$. If we define $\sum_{l=N}^{N-1} = 0$, then the inequality in (10) follows as in the case $i \leq N-1$.

On the other hand, according to Lemma 1, for $x \neq 0$, we have

$$0 < V(x) \leq a_1\,\|x\|_2 + (a_2 + \lambda_{\max}(Q))\,\|x\|_2^2\,. \tag{11}$$

Without loss of generality, we discuss two cases.

Case 1: Assume that $0 < \|x\|_2 \leq 1$. Then $\|x\|_2^2 \leq \|x\|_2$ and hence

$$V(x) \leq (a_1 + a_2 + \lambda_{\max}(Q))\,\|x\|_2\,.$$

According to the formula in (10), we obtain:

$$\begin{aligned} V(f^i(x)) &\leq V(x) - \lambda_{\min}(Q)\,\|x\|_2^2 + \epsilon \\ &\leq \left(1 - \frac{\lambda_{\min}(Q)\,\|x\|_2}{V(x)}\right) V(x) - \lambda_{\min}(Q)\left(\|x\|_2^2 - \|x\|_2\right) + \epsilon \\ &\leq \rho V(x) + \varepsilon\lambda_{\min}(Q) + \epsilon, \end{aligned}$$

where

$$\rho \triangleq 1 - \frac{\lambda_{\min}(Q)\,\|x\|_2}{a_1 + a_2 + \lambda_{\max}(Q)},$$

ε is a positive constant parameter. According to the fact that $0 < \lambda_{\min}(Q) \leq \lambda_{\max}(Q)$, $a_1 > 0$ and $a_2 > 0$, it follows that $0 < \rho < 1$.

Case 2: Assume that $\|x\|_2 > 1$, then $\|x\|_2 < \|x\|_2^2$ and according to (11), we have

$$V(x) < (a_1 + a_2 + \lambda_{\max}(Q)) \|x\|_2^2 .$$

According to the formula in (10), one have

$$\begin{aligned} V(f^i(x)) &\leq \left(1 - \frac{\lambda_{\min}(Q) \|x\|_2^2}{V(x)}\right) V(x) + \epsilon \\ &\leq \rho V(x) + \epsilon \leq \rho V(x) + \varepsilon\lambda_{\min}(Q) + \epsilon. \end{aligned}$$

If $x = 0$, then above inequality still holds, because $V(0) = 0$. Thus, combining Case 1 and Case 2, we have the following inequality

$$V(f^i(x)) \leq \rho V(x) + \varepsilon\lambda_{\min}(Q) + \epsilon. \tag{12}$$

Furthermore, we fix $i \in \mathbb{N}_0$ and note that at time instant k_i, the control packet is successfully transferred to be buffered. Then until the next packet is received at time k_{i+1}, m_i consecutive packet-drops occur. By the PPC strategy, the control input becomes $u(k_i + l) = u_l(x(k_i))$, $l = 1, 2, ..., m_i$, and the states $x(k)$, $k = k_i + 1, ..., k_i + m_i$, are determined by these open-loop controllers. By Assumption 1, we have $m_i \leq N - 1$, then, according to the formula in (12), we obtain:

$$V(x(k)) \leq \rho V(x(k_i)) + \epsilon + \varepsilon\lambda_{\min}(Q), \tag{13}$$

for $k \in \left\{ \begin{array}{cccc} k_i + 1 & k_i + 2 & ... & k_i + m_i \end{array} \right\}$, and also for $k_{i+1} = k_i + m_i + 1$, we have

$$V(x(k_i)) \leq \rho V(x(k_i)) + \epsilon + \varepsilon\lambda_{\min}(Q).$$

Then, it is easy to see that

$$V(x(k_i)) \leq \rho^i \phi \|x(k_0)\|_2 + \frac{1}{1-\rho} (\epsilon + \varepsilon\lambda_{\min}(Q)) . \tag{14}$$

Combining the formulas in (13) and (14), we have

$$V(x(k)) \leq \rho^{i+1} \phi (\|x(k_0)\|_2) + \frac{1}{1-\rho} (\epsilon + \varepsilon\lambda_{\min}(Q)) ,$$

for $k \in \left\{ \begin{array}{cccc} k_i+1 & k_i+2 & \dots & k_{i+1}-1 \end{array} \right\}$, and this inequality holds also for $k = k_{i+1}$. Finally, by using the lower bound of $V(x)$ provided in Lemma 1, we have

$$\|x(k)\|_2 \leq \sqrt{\frac{V(x(k))}{\lambda_{\min}(Q)}} \leq \rho^{\frac{i+1}{2}} \sqrt{\frac{\phi(\|x(k_0)\|_2)}{\lambda_{\min}(Q)}} + \Lambda.$$

This completes the proof. □

Remark 1: According to Theorem 2, the controller can be obtained by using FISTA algorithm [24] or OMP algorithm [25].

In this section, we present the sparse PPC method to reduce the effect of packet drops in the forward channel. Also the sensing signals in feedback channel still need to pass through the unreliable network. If packet drops happen, the control system would be unstable. The following section will address this problem.

IV. Compressed Sensing in Feedback Channel

In this section, we will use CS method in the feedback channel of control systems. In [16], it develops a sequential CS framework based on sliding window processing to deliver the jointly correlated sensor data streams. For networked control systems, the feedback signals also need to go through the unreliable network. Thus, we need address the data transmission problem in the following subsections.

A. Data Structure and Performance Metrics of Compressed Sensing Reconstruction

Before proceeding further, the following definitions are presented for later development.

Definition 1: Compression ratio ρ represents the level of compression:

$$\rho = \frac{M}{N},$$

where N is the size of data transmission before using CS method, and M is the size of transferred data after using CS method.

Definition 2: Signal to noise ratio (SNR) is used to evaluate the reconstructed signal's quality:

$$SNR = 10 \log_{10} \sum_{i=1}^{N} \frac{x_i^2}{(\hat{x}_i - x_i)^2},$$

where the noise is defined by the difference between the original signal (x_i) and the reconstructed signal ($\hat{x}_i$).

Definition 3: Root Mean Square Reconstruction Error ($RMSE$) represents the average value of squared reconstruction error or loss in signal's quality:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \hat{x}_i)^2},$$

where N is the number of samples or length of the signal, x_i is the original signal, and $\hat{x}_i$ is the reconstructed signal from the compressed measurements using a recovery algorithm.

Definition 4: Mean Absolute Error (MAE):

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|x_i - \hat{x}_i|.$$

where N is the number of samples or length of the signal, x_i is the original signal, and $\hat{x}_i$ is the reconstructed data. MAE is often a preferred criterion for time series comparison. It measures the average magnitude of the reconstruction errors, which assigns relatively higher weights to large errors than $RMSE$. MAE follows linear scoring or assigns equal weights to reconstruction error at each example.

Definition 5: Maximum Error Deviation (MED):

$$MED = \max_{i=1,...,N}|x_i - \hat{x}_i|.$$

where N is the number of samples or length of the signal, x_i is the original signal, and $\hat{x}_i$ is the reconstructed data. MED accounts for the worst case error encountered at any time sample.

As Fig. 1, we define that $\bar{x}_1(k)$, $\bar{x}_2(k), ..., \bar{x}_n(k)$ are the sensor signals, which need to be transferred through the unreliable network. Our purpose is to transfer the data $\bar{x}_i(k), i = 1, 2, ..., n$ through the unreliable network and get $\hat{x}_i(k), i = 1, 2, ..., n$. Using CS method, we can reduce the size of transferred signals. Also even some data is dropped, the signals can still be reconstructed perfectly. Thus, let $\bar{X}(k) \in \mathbb{R}^{n\times W}$ denote a data window at time instant k with window size $W \geq 1$. It consists of W consecutive readings of all n sensors at time instants $\{k - W + 1, ..., k\}$ as

$$\bar{X}(k) = \begin{bmatrix} \bar{x}_1(k-W+1) & \cdots & \bar{x}_1(k) \\ \vdots & \ddots & \vdots \\ \bar{x}_n(k-W+1) & \cdots & \bar{x}_n(k) \end{bmatrix},$$

where $\bar{x}_i(k)$ is the reading of sensor $i = 1, 2, ..., n$ at time step k. We define the i^{th} row of $\bar{X}(k)$ is as follows:

$$\tilde{x}_i^T(k) \triangleq \left[\begin{array}{ccc} \bar{x}_i\left(k - W + 1\right) & \dots & \bar{x}_i(k) \end{array} \right],$$

which contains the data of the i^{th} sensor at time instants $\left\{ \begin{array}{ccc} k - W + 1 & \dots & k \end{array} \right\}$. And also define

$$\check{x}(k) \triangleq \left[\begin{array}{ccc} \bar{x}_1(k) & \dots & \bar{x}_n(k) \end{array} \right]^T,$$

as the sensors' readings at a time instant step k. Then, we can describe $\bar{X}(k)$ as

$$\bar{X}(k) = \left[\begin{array}{ccc} \check{x}(k - W + 1) & \dots & \check{x}(k) \end{array} \right] = \left[\begin{array}{ccc} \tilde{x}_1(k) & \dots & \tilde{x}_n(k) \end{array} \right]^T.$$

Assume that there exist a basis $\Psi_S \in \mathbb{R}^{n \times n}$ for the spatial domain and also a basis $\Psi_T \in \mathbb{R}^{W \times W}$ for the temporal domain. Then, each column of $\bar{X}(k)$ has a compressible representation:

$$\check{x}(k) = \Psi_S \theta_S(k),$$

where $\theta_S(k) \in \mathbb{R}^n$ contains the spatial transform coefficients at slot k. Also each row of $\bar{X}(k)$ has a compressible representation:

$$\tilde{x}_i(k) = \Psi_T \theta_{T,i}(k),$$

where $\theta_{T,i}(k) \in \mathbb{R}^W$ contains the temporal transform coefficients of sensor i. Then we can rewrite $\bar{X}(k)$ as

$$\begin{aligned} \bar{X}(k) &= \left[\begin{array}{ccc} \check{x}\left(k - W + 1\right) & \dots & \check{x}(k) \end{array} \right], & (15) \\ &= \Psi_S \left[\begin{array}{ccc} \theta_S(k - W + 1) & \dots & \theta_S(k) \end{array} \right] = \Psi_S \Theta_S(k), \\ \bar{X}(k) &= \left[\begin{array}{ccc} \tilde{x}_1(k) & \dots & \tilde{x}_n(k) \end{array} \right]^T = \left[\begin{array}{ccc} \theta_{T,1}(k) & \dots & \theta_{T,n}(k) \end{array} \right]^T \Psi_T^T = \Theta_T^T(k) \Psi_T^T. & (16) \end{aligned}$$

Kronecker sparsifying basis can succinctly combine the individual sparsifying basis of each signal dimension into a single transformation matrix. Thus, we can merge the transformations in (15) and (16), and represent $\bar{X}(k)$ as

$$\begin{aligned} X(k) &= \text{vec}(\bar{X}(k)) = \text{vec}\left(\Psi_S \Theta_S(k)\right) & (17) \\ &= \text{vec}\left(\Psi_S Z(k) \Psi_T^T\right) \\ &= \left(\Psi_T \otimes \Psi_S\right) \text{vec}(Z(k)) = \Psi z(k), \end{aligned}$$

where $X(k) = \left[\begin{array}{ccc} \check{x}^T(k-W+1) & \dots & \check{x}^T(k) \end{array} \right]^T \in \mathbb{R}^{nW}$ is the vector-reshaped data window, $\Psi = (\Psi_T \otimes \Psi_S) \in \mathbb{R}^{nW \times nW}$ is the Kronecker sparasifying basis, and $z(k) = \mathrm{vec}(Z(k)) = \mathrm{vec}\left(\Theta_S(k)\Psi_T^{-T}\right) \in \mathbb{R}^{nW}$ contains the joint transformation domain coefficients. Note that $Z(k)$ can be interpreted as the matrix representation of spatial domain coefficients $\Theta_S(k)$ in temporal basis Ψ_T, we have $\Theta_S^T(k) = \Psi_T Z^T(k)$. Then, $X(k)$ has a 2D-separable transform $Z(k) = \Psi_S^{-1} X(k) \Psi_T^{-T}$, where Ψ_S^{-1} operates on the columns of $X(k)$, and Ψ_T^{-T} on its rows.

Remark 2: In this method, the choice of the basis Ψ_T and Ψ_S are very important. We can choose them as Fourier transformation basis, Discrete Cosine transformation basis, or Wavelet basis, etc, which depends on the signal's property.

B. Compressed Sensing Encoding

Let us discuss the CS encoding process. At each time instant $k \geqslant 1$, the measurements are taken with respect to the current sensors' readings $\check{x}(k)$. Thus, the sink acquires $M(k)$ linear CS measurements $\tilde{y}(k) = \left[\begin{array}{ccc} \bar{y}_1(k) & \dots & \bar{y}_{M(k)}(k) \end{array} \right]^T \in \mathbb{R}^{M(k)}$ as

$$\tilde{y}(k) = \tilde{\Phi}(k)\check{x}(k), \ k \geq 1, \tag{18}$$

where $\tilde{\Phi}(k) \in \mathbb{R}^{M(k) \times n}$, $M(k) < n$, is the measurement matrix for time instant k. Considering (18), the measurement ensemble with respect to each data window $X(k)$ has the following block-diagonal structure:

$$\begin{bmatrix} \tilde{y}(k-W+1) \\ \vdots \\ \tilde{y}(k) \end{bmatrix} = \begin{bmatrix} \tilde{\Phi}(k-W+1) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \tilde{\Phi}(k) \end{bmatrix} \begin{bmatrix} \check{x}(k-W+1) \\ \vdots \\ \check{x}(k) \end{bmatrix}. \tag{19}$$

Accordingly, by forming the measurement vector $y(k) = \left[\begin{array}{ccc} \tilde{y}^T(k-W+1) & \dots & \tilde{y}^T(k) \end{array} \right]^T \in \mathbb{R}^{\sum_{\tau=k-W+1}^{k} M(\tau)}$, and the block-diagonal measurement matrix

$$\bar{\Phi}(k) = \mathrm{diag}\left(\begin{array}{ccc} \tilde{\Phi}(k-W+1) & \dots & \tilde{\Phi}(k) \end{array} \right) \in \mathbb{R}^{\sum_{\tau=k-W+1}^{k} M(\tau) \times nW},$$

the measurement ensemble in (19) can be compactly written as

$$y(k) = \bar{\Phi}(k) X(k). \tag{20}$$

Note that the measurement matrices $\tilde{\Phi}(k) \in \mathbb{R}^{M(k) \times n}$, $k = 1, 2, ...$, have different structures and varying number of measurements $M(k)$.

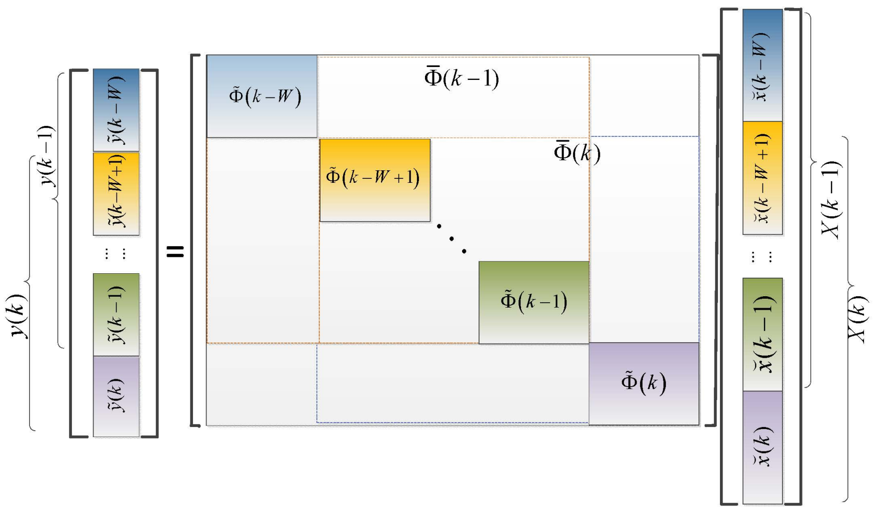

Fig. 2. Illustration of the sliding window processing with respect to sensing data and transfer data

Remark 3: As Fig. 2 shows, in the traditional method, the data $\breve{x}(k-W+1), ..., \breve{x}(k)$ are transferred through the unreliable networks, and the transfer data size of $\breve{x}(k)$ is n at time instant k. Now, using the CS encoding, we only need to transfer $\tilde{y}(k-W+1), ..., \tilde{y}(k)$ instead of the data $\breve{x}(k-W+1), ..., \breve{x}(k)$. The transfer data size of $\tilde{y}(k)$ is $M(k)$, which is usually much smaller than n.

C. Compressed Sensing Decoding

By exploiting the joint spatio-temporal compressibility (17), each data window $\bar{X}(k)$ can be recovered from measurements (20) by solving the l_1-minimization problem

$$\hat{z}(k) := \arg\min_{\tilde{z}} \|z\|_1 \,, \ \text{s.t. } y(k) = \bar{\Phi}(k)\Psi z, \tag{21}$$

reconstructing $\hat{x}(k) = \Psi\hat{z}(k)$, and reshaping it as $\hat{X}(k) = \left[\begin{array}{ccc} \hat{x}(k-W+1) & \dots & \hat{x}(k) \end{array} \right]$, where $\hat{x}(k) = \left[\begin{array}{ccc} \hat{x}_1(k) & \dots & \hat{x}_n(k) \end{array} \right]^T$ are the estimates of the sensors' readings at time instant k. Thus, each decoding instant (21) produces estimates for the current sensors' readings and the $W-1$ previous ones.

Remark 4: Fig. 2 shows that every time when obtaining the current time $\tilde{y}(k)$, we combine $\tilde{y}(k-1), ..., \tilde{y}(k-W+1)$, and reconstruct the signal $\breve{x}(k-W+1), ..., \breve{x}(k)$. Then, we obtain feedback data $\breve{x}(k)$. The method has the following advantages. First, the transfer size of sensory data is reduced. Second, since the transfer data size is reduced, it can reduce network congestion and thus reduce the delay of data transfer.

Remark 5: It is worth noting that the effectiveness of the approach depends heavily on the choices of bases for the temporal and spatial compression. For a control system, the signals always go through the facilities like plants, sensors and so on. Since these facilities serve as low-pass filters for signals, the high frequency part of the signals can not pass through these facilities. Thus, the signals are always sparse in Fourier basis and discrete cosine basis and even the discrete wavelet basis, which can result in a more compact representation.

V. Illustrative Example

In this section, we consider two examples to demonstrate the effectiveness of the proposed methods.

Example 1: We consider a networked control of a mechanical system as Fig. 3. In the system, M_1 and M_2 is the mass of the cars and also represent the cars in the following discussion. y_1 and y_2 is the displacement of M_1 and M_2 respectively. k is stiffness of the spring, and b is damped coefficient. Traditionally, to simplify the problem, b is always assumed to be constant. But in practice, b always changes by temperature and other environment conditions. Thus, for accurate description of this system, we assume that the damped coefficient b always changes in two modes ($b_1 = 0.6Nm/s,\ b_2 = 0.3Nm/s$). All of the surface frictions are ignored, and F is an external force and also a control input signal in the system. Set $M_1 = 1kg,\ M_2 = 2kg$, $k = 10N/m$ and the sampling period $T = 10ms$. According to Newton's mechanics laws, and we get the discrete model of the networked switched system as follows:

$$x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k)$$

where

$$A_1 = \begin{bmatrix} 0.9991 & 0.0009 & 0.01 & 0 \\ 0.0004 & 0.9996 & 0 & 001 \\ -0.1791 & 0.1791 & 0.9931 & 0.0069 \\ 0.0896 & -0.0896 & 0.0034 & 0.9966 \end{bmatrix},\ A_2 = \begin{bmatrix} 0.9991 & 0.0009 & 0.01 & 0 \\ 0.0004 & 0.9996 & 0 & 001 \\ -0.0896 & 0.0896 & 0.9931 & 0.0069 \\ 0.0896 & -0.0896 & 0.0034 & 0.9966 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0.01 & 0 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 & 0 & 0.01 & 0 \end{bmatrix}.$$

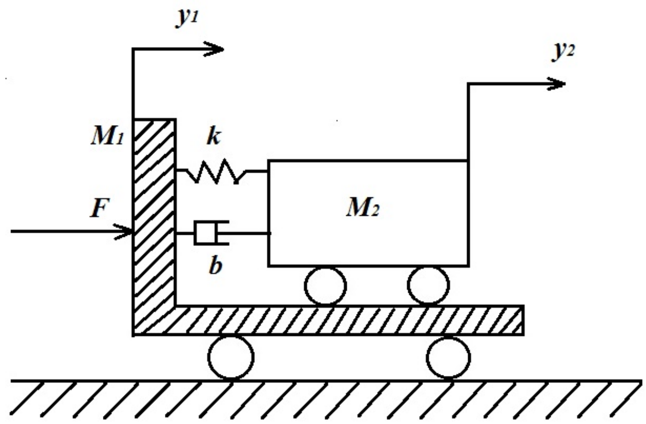

Fig. 3. The mechanical system

Set $x(0) = [1,1,1,1]$, $N = 5$, $\mu = 10.7167$ is a punishment coefficient of the control input u_i in (5). Set $\epsilon = 28$, and we have $r = 4.1042$ according to Theorem 2. Choose $Q = I$, and according to Theorem 1, we get

$$P_1 = \begin{bmatrix} 133.71 & 116.92 & 79.96 & -24.85 \\ * & 132.15 & 86.31 & -27.17 \\ * & * & 347.91 & -160.18 \\ * & * & * & 90.64 \end{bmatrix}, \ P_2 = \begin{bmatrix} 133.80 & 116.84 & 79.30 & -24.53 \\ * & 132.20 & 86.89 & -27.46 \\ * & * & 346.19 & -159.29 \\ * & * & * & 80.17 \end{bmatrix}.$$

Then, the sparse controller can be obtained as form of (4) by Theorem 2. Fig. 4 shows that the open-loop system is not stable. Fig. 5 shows that the closed-loop system with the sparse controller in (4) is stable. Fig. 6 shows the situation of packet drop $d(k)$. For comparison, we also synthesize PPC with the conventional l_2 cost function:

$$J_2(U, x(k)) \triangleq \|x(N|k)\|^2_{P_{\sigma(k)}} + \sum_{i=0}^{N-1} \|x(i|k)\|^2_Q + \mu \sum_{i=0}^{N-1} \|u_i\|_2 \, . \tag{22}$$

Using the proposed method with the cost function in (5) and the method with the cost functions in (22), we can get two groups of controller measurements $u(t)$ with l_1/l_2 gain controller and l_2 gain controller. The control value obtained by l_1/l_2 optimization include 52 zero values of 1000 data, whereas the conventional l_2 ones contain only 30 zeros values.

Example 2: In this example, we use a load frequency control model to demonstrate the effectiveness of the CS method in feedback control channel. We linearize the power system

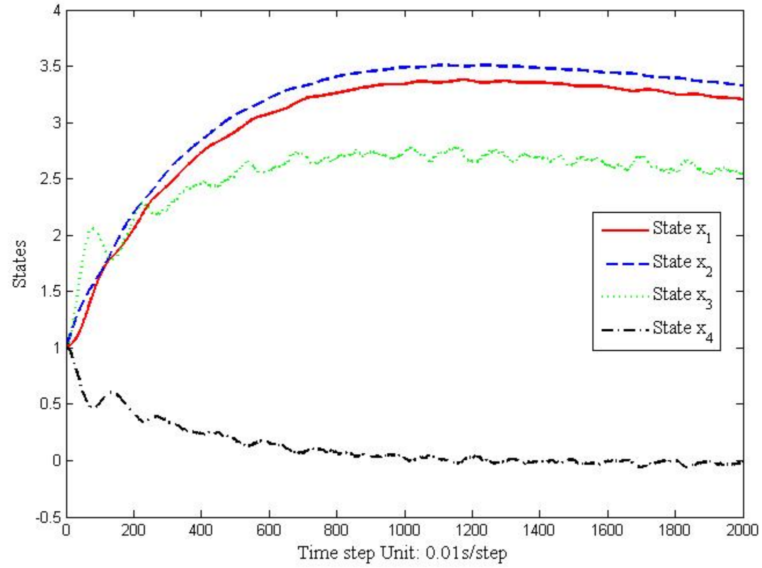

Fig. 4. The states of open-loop networked switched systems

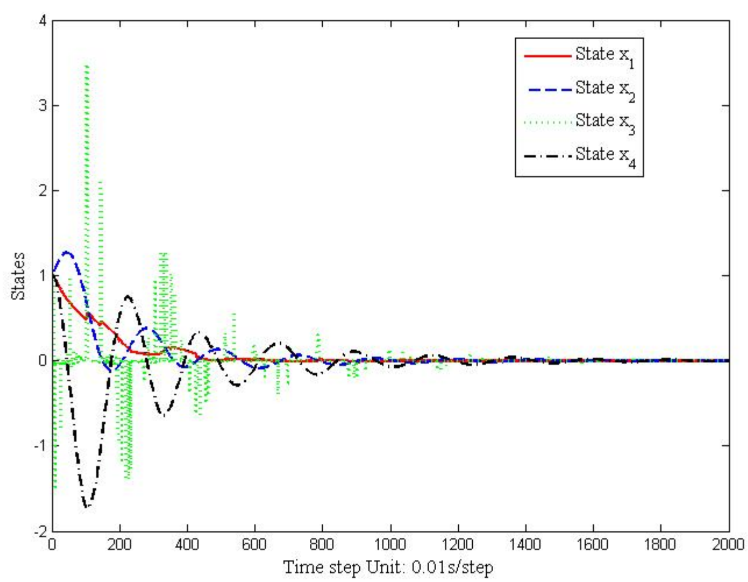

Fig. 5. The states of closed-loop networked switched systems with l_1/l_2 gain controller.

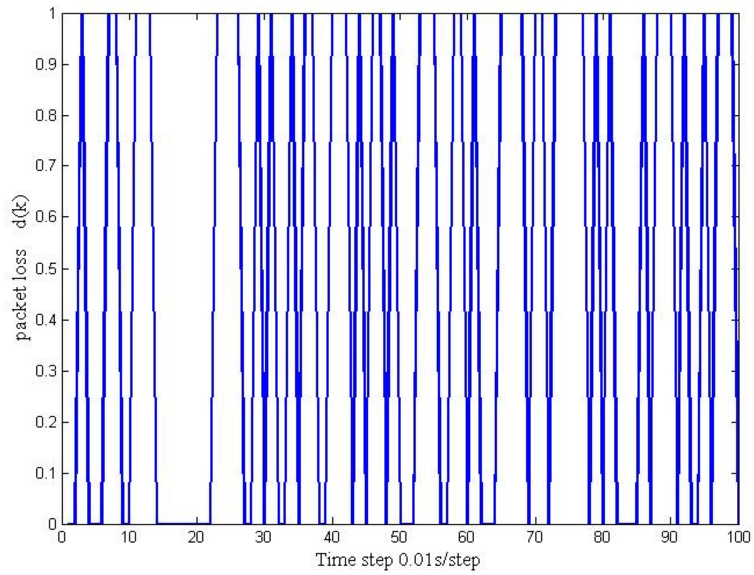

Fig. 6. The packet loss of networked switched systems.

as the example in [26]. The range of system parameter variations in this example problem is selected as follows:

$$\begin{aligned}
\frac{1}{T_{T_1}} &\in \left[\begin{array}{cc} 2.778 & 4.167 \end{array}\right], \ \frac{1}{T_{G_1}} \in \left[\begin{array}{cc} 10.417 & 15.625 \end{array}\right], \\
\frac{1}{R_1 T_{G_1}} &\in \left[\begin{array}{cc} 3.617 & 8.138 \end{array}\right], \ \frac{1}{T_{T_2}} \in \left[\begin{array}{cc} 2.525 & 3.788 \end{array}\right], \\
\frac{1}{T_{G_2}} &\in \left[\begin{array}{cc} 11.574 & 17.361 \end{array}\right], \ \frac{1}{R_2 T_{G_2}} \in \left[\begin{array}{cc} 3.968 & 8.929 \end{array}\right], \\
\frac{1}{T_{T_3}} &\in \left[\begin{array}{cc} 2.381 & 3.571 \end{array}\right], \ \frac{1}{T_{G_3}} \in \left[\begin{array}{cc} 11.905 & 17.857 \end{array}\right], \\
\frac{1}{R_3 T_{G_3}} &\in \left[\begin{array}{cc} 3.968 & 8.929 \end{array}\right].
\end{aligned}$$

The nominal plant models of the system for the three areas are considered in this example. We assume that there are no parameter uncertainty and load disturbances, and obtain the state-feedback gains as:

$$\begin{aligned} K_1 &= \begin{bmatrix} -39.71 & -19.60 & -0.69 & 0.95 & -31.84 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} 18.26 & 29.12 & 6.12 & -11.62 & -3.18 \end{bmatrix}, \\ K_3 &= \begin{bmatrix} -34.00 & -16.94 & -0.45 & 0.96 & -26.33 \end{bmatrix}. \end{aligned} \tag{23}$$

For the parameters in the nominal plant models of three areas, Fig. 7 shows that the closed-loop system is stable. And also there are 15 states in the systems. Assume that all of the states information needs to be transferred through the unreliable network. Table I shows that the CS result in different metrics with different compressed ratio by using DCT basis. The $10\ dB$ SNR value is commonly used in the literature as a quality threshold [27]. In Table I, when $\rho = \frac{6}{15} = 40\%$, the $SNR = 12.35$. Then using the CS method in last section, we transfer and reconstruct the sensing signals of control the systems with $\rho = 40\%$. Fig. 8 shows that the by CS method, the closed-loop system is still stable, which demonstrates the effectiveness of the presented method.

Remark 6: To recover data in an unreliable network, we only need to increase compression ratio accordingly. For example, we set nominal $\rho = \frac{10}{15}$, which means that the size of $\tilde{y}(k)$ is 10. According to the CS theory, even if only 6 data of $\tilde{y}(k)$ is received, we can still reconstruct the whole data $\check{x}(k)$ (since the actual $\rho = 6/15 = 40\%$). Thus, the method is demonstrated to have strong robustness in the control system feedback data transmission.

VI. Conclusions

This paper has investigated the problems of PPC controller design and data transmission for networked switched control systems. Considering the model of networked switched systems, the PPC control method has been presented, and also the stable theorem has been proposed and proven. Then, considering the feedback channel, sensors' data is transferred through the unreliable networks by CS method. Illustrative examples are used to demonstrate the effectiveness of the proposed controller design method and the data transmission method. The proposed CS method offers three main benefits. First, it can reduce the size of transfer data. Second, if packet drop happens, the data can still be recovered in high probability. Third, the energy for transferring data is reduced since less amount of data needs to be transferred. In future work, we

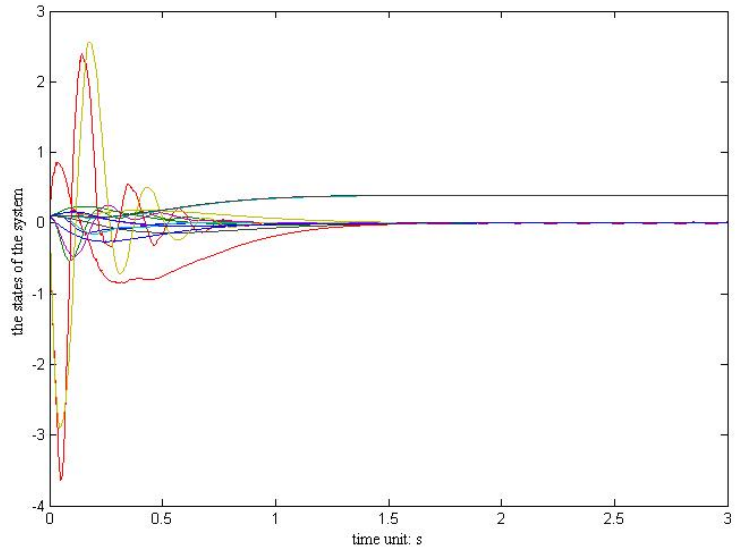

Fig. 7. The state of closed-loop system with all data transferred.

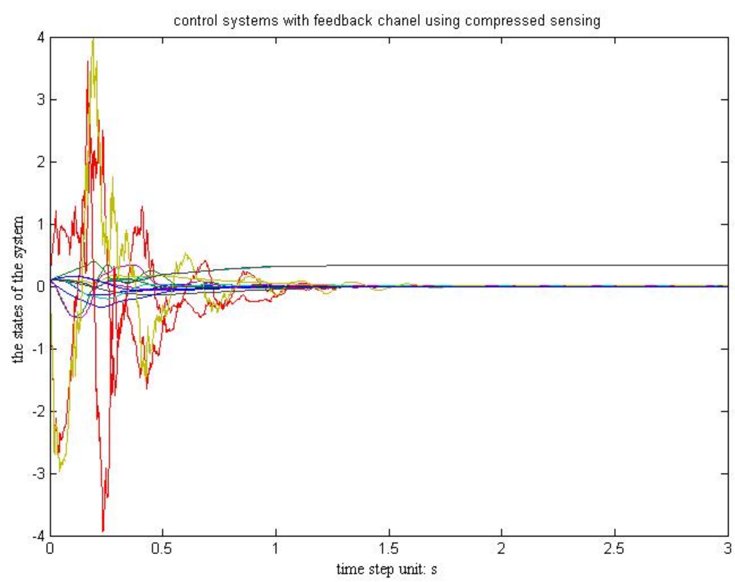

Fig. 8. The states of closed-loop sysstem with only 40% data size transferred by CS.

TABLE I

SOME PERFORMANCE METRICS FOR DIFFERENT COMPRESSION RATIO ρ

M	ρ	SNR	$RMSE$	MAE	MED
12	$\frac{12}{15}$	30.79	0.0043	0.0013	0.0961
11	$\frac{11}{15}$	25.67	0.0066	0.0019	0.2167
10	$\frac{10}{15}$	22.22	0.0098	0.0025	0.2723
9	$\frac{9}{15}$	20.79	0.0111	0.0029	0.2753
8	$\frac{8}{15}$	17.58	0.0161	0.0043	0.4975
7	$\frac{7}{15}$	15.72	0.0225	0.0062	0.5821
6	$\frac{6}{15}$	12.35	0.0296	0.0079	0.8988
5	$\frac{5}{15}$	9.83	0.0378	0.0097	0.8467

will investigate more complex networked system model such as time delay networked system model, and also we will consider the impact from the practical environment factors.

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