Automata Qual Exam (Spring 2011)
Answer ALL questions (Closed Book Exam)

Question 1 (20 points)

Consider a variant of the nondeterministic finite automata (NFA) model. For this model, all the states of an NFA are prioritized such that the priorities of the states form a linear ordering. After all symbols of an input are read, the automaton may arrive at a subset of states Q'. The automaton accepts the input iff the state with the highest priority among the states in Q' is an accepting state. Does this new model of NFA still recognize regular languages? Justify your answer.

Question 2

We define "logic tree" to be a binary tree with each leaf node labeled by a truth value **t** (true) or **f** (false), internal node with one child labeled by $\neg$, and internal node with two children labeled by $\vee$ or $\wedge$. The semantics of a logic tree t is a truth value $v(t)$, called value of tree t, defined as follows: if the logic tree has only a leaf node, then $v(t)$ is the label of the leaf node; if the root node has the label $\neg$ and t' is the subtree of the root, then $v(t) = \neg v(t')$; otherwise, let t' and t'' be the two subtrees of t, and $v(t) = v(t') \vee v(t'')$ (or, $v(t') \wedge v(t'')$) if the root node has the label $\vee$ (or, $\wedge$).

We define a preorder traversal of a logic tree as follows:

```
preorder( t ) {
   print the label of the root node of t

   if the root node of t has only one child
      then // let t' be the child of the root node
           { preorder(t'); }
   else if root node of t has two children
      then // let t' be the left  child of the root node
           // let t" be the right child of the root node
           { preorder(t'); preorder(t"); }
}
```

Let t be a logic tree. We write $< t >$, called the prefix code of t, to denote the output obtained from the preorder traversal of t.

(a) (10 points) Show that two different logic trees cannot have the same prefix code.

Let L be the language consists of prefix codes of logic trees t such that the value of t is true. That is, $L = \{ \ < t > \ \mid v(t) = \text{true} \ \}$.

(b) (20 points) Give a context-free grammar for L.

We define a postorder traversal of a logic tree as follows:

```
preorder( t ) {
   if the root node of t has only one child
      then // let t' be the child of the root node
           { postorder(t'); }
   else if root node of t has two children
      then // let t' be the left  child of the root node
           // let t" be the right child of the root node
           { postorder(t'); postorder(t"); }

   print the label of the root node of t
}
```

Let t be a logic tree. We write $[t]$, called the postfix code of t, to denote the output obtained from the postorder traversal of t.

Let L' be the language consists of postfix codes of logic trees t such that the value of t is true. That is, $L = \{ \ [t] \mid v(t) = \text{true} \ \}$.

(c) (20 points) Give a pushdown automaton for L'. You are **encouraged** to describe the automaton using plain English.

Question 3 (15 points + 15 points)

Let C be a language. Prove that C is Turing-recognizable (recursively enumerable) iff a Turing-decidable (recursive) language D exists such that $C = \{x \mid \exists y \ < x, y > \in D\}$.