Department of Computer Science Spring 2011 New Mexico State University

Ph.D. Qualifying Exam: Analysis of Algorithms

This is a closed book exam. The total score is 100 points. Please answer all questions.

1. This question is concerned with partitioning the indices $\{1,2,\ldots,n\}$ of n given integers $a_1,\ldots,a_n$ into disjoint subsets. Let S be the summation of all numbers, i.e.,

$$S = \sum_{i=1}^{n} a_i$$

(30 points) (a) **The 2-Partition problem.** Determine whether it is possible to partition $\{1,\ldots,n\}$ into 2 disjoint subsets I, J such that

$$\sum_{i\in I} a_i = \sum_{j\in J} a_j = \frac{1}{2}S$$

For example, for input $(2,3,8,5,4)$, the answer is yes, because there is the partition $(3,8)$, $(2,5,4)$ whose sums are all 11. On the other hand, for the input $(3,2,2,5)$ the answer is no.

Design a dynamic programming algorithm to solve this problem in $O(nS)$ time. Give the subproblems definition, the recurrence, and the base cases for the dynamic programming.

Hint: check to see if some of the first i numbers can add up to some integer s.

(30 points) (b) **The 3-Partition problem.** Determine whether it is possible to partition $\{1,\ldots,n\}$ into 3 disjoint subsets I, J, K such that

$$\sum_{i\in I} a_i = \sum_{j\in J} a_j = \sum_{k\in K} a_k = \frac{1}{3}S$$

For example, for input $(2,1,3,8,5,4,4)$, the answer is yes, because there is the partition $(1,8)$, $(4,5)$, $(2,3,4)$ which sums are all 9. On the other hand, for the input $(3,2,2,5)$ the answer is no.

Design a dynamic programming algorithm for 3-Partition. Give the subproblems definition, the recurrence, and the base cases. Your algorithm should run in time polynomial in n and S^2.

(40 points) 2. Design a linear algorithm to find the number of shortest paths between two given nodes u and v in an undirected graph $G = (V,E)$, by modifying the Breadth-First-Search algorithm. We assume all edges have the same distance of 1.