Automata Qual Exam (Spring 2011)
Answer ALL questions (Closed Book Exam)

Question 1

We consider the concept of Queue Automata which makes use of a queue. (In contrast, a PDA uses a stack.)

Assume that there are four queue operations: `isEmptyQueue`, `front`, `enqueue`, and `dequeue` where `front` returns the front element without removing it, while `dequeue` removes the front element.

(a) (10 points) Give a formal definition of a Queue Automaton. Specifically, define the transition function in detail, explain how the new configuration (instantaneous description) is computed, and define when a string is accepted.

(b) (15 points) Explain how a Turing Machine can be simulated by a Queue Automaton.

Question 2

Given two DFAs (deterministic finite automata) $M = (P, \Sigma, \delta, p_0, F)$ and $N = (Q, \Sigma, \delta', q_0, F')$. We consider constructing a new DFA that simulates simulaneously both DFAs in parallel. (Some books may call this the "cartesian" product of two DFA.)

(a) (5 points) Give a formal definition of the cartesian product of M and N.

(b) (10 points) Explain how one can decide if two given DFAs are equivalent (in that both recognize the same language) by inspecting the cartesian product of the two DFAs.

Question 3 (20 points)

Consider the following grammar:

exp $\longrightarrow$ (exp) | exp + exp | exp $*$ exp | exp $\hat{}$ exp | 0 | 1

Suppose + and $*$ are left associative; but $\hat{}$ is right associative. Also, suppose $\hat{}$ is of higher precedence than $*$, which again is of higher precedence than +. Re-write the grammar so that it reflects the intended meaning given, and is unambiguous.

Question 4

Consider the pumping lemma for context-free languages. In the textbooks, the proof of the pumping lemma is explained with reference to parse trees for strings that are long enough.

(a) (20 points) Given a specific parse tree, explain how one can determine if the parse tree can be "pumped". That is, give a syntactic property that characterizes "pump-able" parse trees.

(b) (20 points) Consider a specific configuration (instantaneous description) sequence of the computation of a PDA on an input string. Explain how one can determine if the computation can be "pumped". That is, give a syntactic property that characterizes "pump-able" configuration (instantaneous description) sequence. **Hint:** Translate the syntactic condition from part (a) about parse tress to a condition about configuration (instantaneous description) sequences of a PDA.